AFAPL-TR-78-6
Part II

LEVEL II

(2)

AD A065554

ROTOR-BEARING DYNAMICS TECHNOLOGY DESIGN GUIDE

Part II Ball Bearings

SHAKER RESEARCH CORP.
BALLSTON LAKE, NEW YORK 12019

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TECHNICAL REPORT AFAPL-TR-78-6, Part II
Interim Report for Period April 1976 – October 1977

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AIR FORCE/56780/16 February 1979 — 210

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: AFAPL-TR-78-6-Part II-2	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): Rotor-bearing Dynamics Technology Design Guide Part II, Ball Bearings,	5. TYPE OF REPORT & PERIOD COVERED: Interim Report, April 1976 — October 1977
	6. PERFORMING ORG. REPORT NUMBER: SRC-78-TR-29
7. AUTHOR(s): A. B. Jones, J. M. McGrew, Jr.	8. CONTRACT OR GRANT NUMBER(s): F33615-76-C-2038
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Shaker Research Corporation, Northway 10 Executive Park, Ballston Lake, N. Y. 12019	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 3048 06 85
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Aero Propulsion Laboratory/SFL, Air Force Systems Command, Wright Patterson AFB, Ohio 45433	12. REPORT DATE: February 1978 / 13. NUMBER OF PAGES: 101
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified / 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Ball Bearings
Ball Bearing Stiffness
Turbine Bearings
Rotordynamics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is an update of the original Part IV of the Rotor-bearing Dynamics Design Technology Series, AFAPL-TR-65-45 (Parts I through X). A computer program is given for preparation of ball bearing stiffness data input for rotordynamic response programs. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of ball bearing stiffness.

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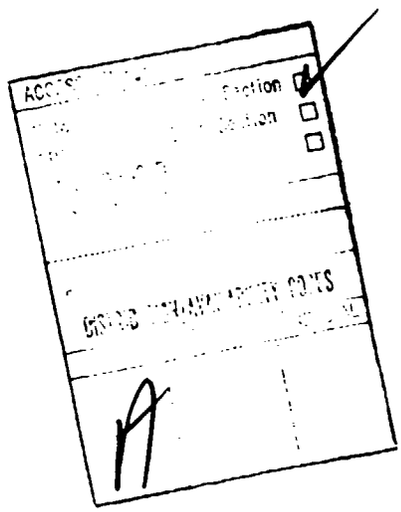
The resulting program is reasonably small and easy to use. Lastly, the stiffness data included in the original Part IV have been updated and are included in appendices.

FORWARD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 304B, "Fuels, Lubrication, and Fire Protection", Task 304806, "Aerospace Lubrication", Work Unit 30480685, "Rotor-Bearing Dynamics Design".

The work reported herein was performed during the period 15 April 1976 to 15 November 1977, under the direction of John B. Schrand (AFAPL/SFL) and Dr. James F. Dill (AFAPL/SFL), Project Engineers. The report was released by the authors in December 1977.



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NOMENCLATURE

Symbol	Description	Units
a	Semi-major axis of pressure ellipse	in.
b	Semi-minor axis of pressure ellipse	in.
B	Total curvature = $f_1 + f_2 - 1$	
B_{ij}	Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.	$\frac{\text{lb-sec}}{\text{in}}$
$\underline{\underline{B}}_N$	Damping matrix $\begin{bmatrix} (\underline{\underline{B}}_N)_{\text{lineal}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & (\underline{\underline{B}}_N)_{\text{angular}} \end{bmatrix}$	
$(\underline{\underline{B}})_{\text{lineal}}$	Damping matrix due to lateral velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{\underline{B}})_{\text{angular}}$	Damping matrix due to angular velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb-sec}}{\text{radian}}$
C	Basic dynamic capacity	lb.

Symbol	Description	Units
d	Ball diameter	in.
D	Diameter	in.
E	Pitch diameter	in.
E'	Operating pitch diameter	in.
E_B	Modulus of elasticity for ball	lbs/in^2
E_R	Modulus of elasticity for race	lbs/in^2
$E(\varepsilon)$	Complete elliptic integral of the second kind formed with the modulus sine where $\cos\varepsilon = b/a$	
f	r/d	
F_i	Applied load in i direction, i = x, y, z	lbs.
F_c	Ball centrifugal load	lbs.
$\underline{F}$	Force matrix = $\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$	lbs.
i	Imaginary number unit vector = $\sqrt{-1}$	
$J_x(\zeta)$	Radial load integral	
K	Ball-race stiffness	lb/in.
K_{ij}	Stiffness component, change of force in i direction due to displacement in j direction. i = x, y, z; j = x, y, z	lb/in.
$\underline{\underline{K}}_N$	Stiffness matrix $\begin{bmatrix} (\underline{\underline{K}}_N)_{lineal} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & (\underline{\underline{K}}_N)_{angular} \end{bmatrix}$	

Symbol	Description	Units
$(\underline{K})_{lineal}$	Stiffness matrix due to lateral displacements	lb/in.
	$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{lineal}$	
$(\underline{K})_{angular}$	Stiffness matrix due to angular rotations	$\frac{\text{in.-lb}}{\text{rad}}$
	$\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{angular}$	
K_z	Axial deflection constant	
$K(\varepsilon)$	Complete elliptic integral of first kind	
M	Applied moment	in.-lb
M_g	Gyroscopic moment	in.-lb
n	Number of balls	
P	Ball load	lb.
P_D	Diametral clearance	in.
Q	Spin torque	in.-lb
r	Raceway groove curvature radius	in.
R	Pitch radius	in.
$\underline{x}$	Column vector = $\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \theta_x \\ \theta_y \end{bmatrix}$	

Symbol	Description	Units
$\underline{W}_N$	Column vector = $\begin{bmatrix} \delta_x \\ \delta_y \\ \theta_x \\ \theta_y \end{bmatrix}$	
$\underline{\underline{Z}}_N$	Impedance matrix = $\underline{\underline{K}}_N + i\,\nu\underline{\underline{B}}_N$	

Symbol	Description	Units
α	Angle between ball rotational axis and center line of bearing	radians, ^{o}
β	Contact angle	radians, ^{o}
β'	Free contact angle	radians, ^{o}
γ	$\frac{d \cos\beta}{E}$	
Γ	Residues of Newton-Raphson equation	
δ	Displacement	in.
δ_x	Lateral displacement in x direction	in.
δ_y	Lateral displacement in y direction	in.
δ_z	Axial displacement	in.
Δ	Ball-race deflection or normal approach	in.
ζ	$= \frac{1}{2}\left[1 + \frac{P_D}{2\,\delta_{max}}\right]$	
ε_i	Hertzian contact parameter; Eqs. (73,88,91,92)	
η_B	$= \frac{4(1-\nu_B^2)}{E_B}$	in.^{2}/lb
η_R	$= \frac{4\,(1-\nu_R)^2}{E_R}$	in.^{2}/lbs
θ_x	Angular rotation about x axis	radians, ^{o}
θ_y	Angular rotation about y axis	radians, ^{o}
θ_z	Angular rotation about z axis	radians, ^{o}
μ	Coefficient of sliding friction	
ν	Frequency of vibration	radians/sec
ν_B	Poisson's ratio for ball	

Symbol	Description	Units
ν_R	Poisson's ratio for race	
ω_B	Rotational velocity of ball	radians/sec
ω_1	Rotational velocity of outer race relative to cage	radians/sec
ω_2	Rotational velocity of inner race relative to cage	radians/sec
ω_s	Ball spin rotational velocity	radians/sec
Ω_E	Absolute orbiting velocity of ball	radians/sec
Ω_1	Absolute rotational velocity of outer race	radians/sec
Ω_2	Absolute rotational velocity of inner race	radians/sec
ϕ	Angular location of ball	radians, ^{o}

SUBSCRIPTS

Symbol	Description
b	Refers to bearing
B	Refers to ball
c	Refers to centrifugal
i	Refers to outer (i = 1) or inner (i = 2) race
ℓ	Refers to loaded extent of bearing
max	Refers to maximum condition
p	Refers to pedestal
q	Refers to ball circumferential position
R	Refers to race
x	Refers to x direction
y	Refers to y direction
z	Refers to z direction
1	Refers to outer race
2	Refers to inner race

SECTION I

INTRODUCTION AND SUMMARY

In recent years, the Rotor-Bearing Dynamics Design Technology Series, AFAPL-TR-65-45 (Parts I through X) has been considered by many engineers to be an important part of their basic analytical tool kit. However, since the issuance of the first volume, in May of 1965, the state-of-the-art has significantly advanced. Further, new techniques of data presentation have been developed, computer capabilities have increased and some minor typographical and technical errors were uncovered.

Part IV of AFAPL-TR-65-45 treated design data for typical deep-groove and angular contact bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of ball bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (3) considers both elastohydrodynamic and cage effects. The more recent computer based design guide prepared by Crecelius and Pirvics (4) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

Thus very sophisticated analytical tools are available for the design and application of ball bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition both computer programs are very large and require an extensive computer facility for use.

The present volume is intended as an update of the original Part IV(1). Those aspects of the original Part IV(1) which treated general design

aspects of ball bearings, load capacity, speed limitations, etc. have been deleted since their coverage is superficial compared to the more sophisticated computer tools now available (3,4). Only those parts directly connected with preparation of input for the rotordynamic response programs (Volume I of the revised series) have been retained. The complete stiffness matrix is calculated including centrifugal effects. Considerations such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of ball bearing stiffness. The resulting program (Appendix A) is reasonably small and easy to use. Lastly, the stiffness data included in the original Part IV have been updated and are included in Appendices B, C and D.

SECTION II

ANALYSIS

2.1 General Bearing Model and Coordinate System

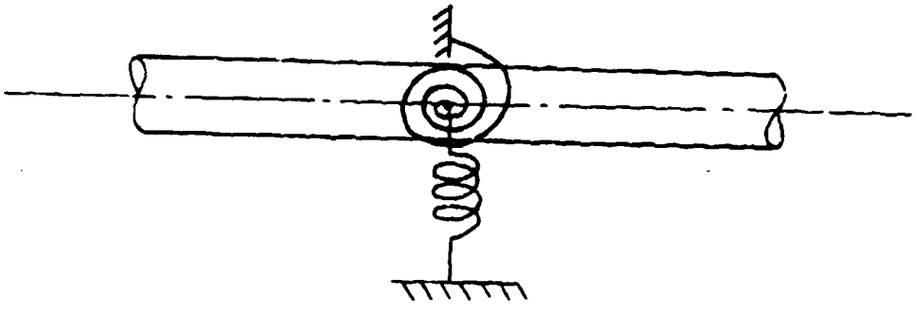
Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- The restraining force/moment may not be temporally in phase with the displacement/inclination.
- The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- Bearing pedestal compliance may not be negligible.

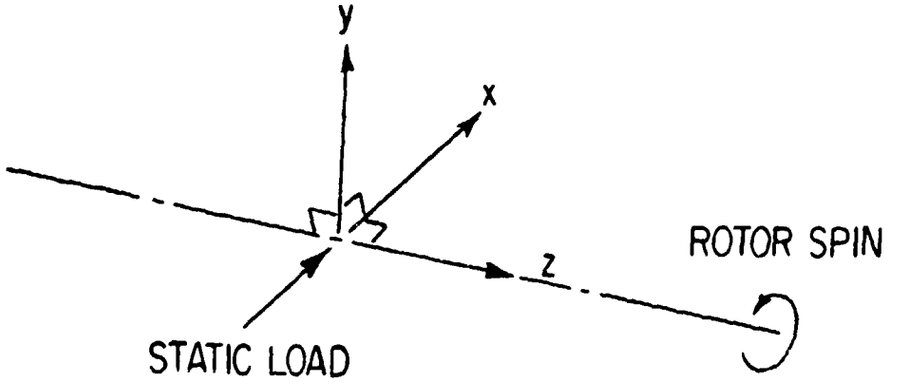
To accommodate the above considerations, the support bearing characteristics are described in Reference 2 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):

$$\underline{R}_N = - \underline{\underline{Z}}_N \cdot \underline{W}_N \qquad (1)$$

where $\underline{W}_N$ is a column vector containing elements which are the two lateral displacements (δ_x, δ_y) and the two lateral inclinations (θ_x, θ_y) of the



(a) Bearing Stiffness Model



(b) Bearing Location
Coordinate System

Figure 1

rotor axis at the bearing station N.

Employing a right handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right handed triad (x, y, z). (δ_x, δ_y) are respectively lateral lineal displacement components of the rotor axis along the (x, y) directions. (θ_x, θ_y) are lateral inclination components respectively in the (z-x, z-y) planes. Note that θ_x is a rotation about the y-axis, while θ_y is a rotation about the negative x-axis.

$\underline{\underline{Z}}_N$ is a complex (4 x 4 matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{\underline{Z}}_N = \underline{\underline{K}}_N + i\nu\underline{\underline{B}}_N \tag{2}$$

where $\underline{\underline{K}}_N$ is the stiffness matrix and $\underline{\underline{B}}_N$ is the damping matrix. ν is the frequency of vibration. Most commonly, lateral linear and angular displacements do not interact with each other so that the non-vanishing portions of $\underline{\underline{K}}_N$ and $\underline{\underline{B}}_N$ are separate 2 x 2 matrices. That is

$$\underline{\underline{K}}_N = \begin{bmatrix} (\underline{\underline{K}}_N)_{\text{lineal}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & (\underline{\underline{K}}_N)_{\text{angular}} \end{bmatrix} \tag{3}$$

$$\underline{\underline{B}}_N = \begin{bmatrix} (\underline{\underline{B}}_N)_{\text{lineal}} & \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} & (\underline{\underline{B}}_N)_{\text{angular}} \end{bmatrix} \tag{4}$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

$$(\underline{\underline{K}})_{\text{lineal}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}} \tag{5}$$

$$(\underline{\underline{B}})_{lineal} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{lineal} \tag{6}$$

$$(\underline{\underline{K}})_{angular} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{angular} \tag{7}$$

$$(\underline{\underline{B}})_{angular} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{angular} \tag{8}$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{\underline{Z}}_N = (\underline{\underline{Z}}_b^{-1} + \underline{\underline{Z}}_p^{-1}) \tag{9}$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{\underline{Z}}_p$.

2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$ in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (θ_x, θ_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

$$\underline{F} = -\underline{\underline{Z}} \cdot \underline{x} \tag{10}$$

$\underline{F}$ is a column vector comprising the five reaction components (F_x, F_y, F_z, M_x, M_y), while $\underline{x}$ is the displacement vector (δ_x, δ_y, δ_z, θ_x, θ_y). $\underline{\underline{Z}}$ is a (5 x 5) matrix containing the elements Z_{ij} with both indicies (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{x} = \underline{x}_o + \underline{x}'; \quad \underline{F} = \underline{F}_o + \underline{F}' \tag{11}$$

($\underline{x}'$, $\underline{F}'$) are respectively presumed to be infinitesimal in comparison with ($\underline{x}_o$, $\underline{F}_o$). Accordingly, Z_{ij} are regarded as dependent on $\underline{x}_o$ but not on $\underline{x}'$. To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.

As illustrated, the load-displacement relationship is a 3/2 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, $\delta'_x < \delta_{x_o}$, the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

$$F'_x = \frac{\partial F_x}{\partial \delta x} \delta'_x \tag{12}$$

where δ'_x is the incremental displacement. $\partial F_x / \partial \delta x$ will depend on the amplitude of δ_{x_o}.

The question of history dependence is resolved by regarding $\underline{x}'$ as periodic motions at any frequency ν of interest, and Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency ν.

To avoid notational clumsiness, the primes will be dropped from ($\underline{F}'$, $\underline{x}'$)

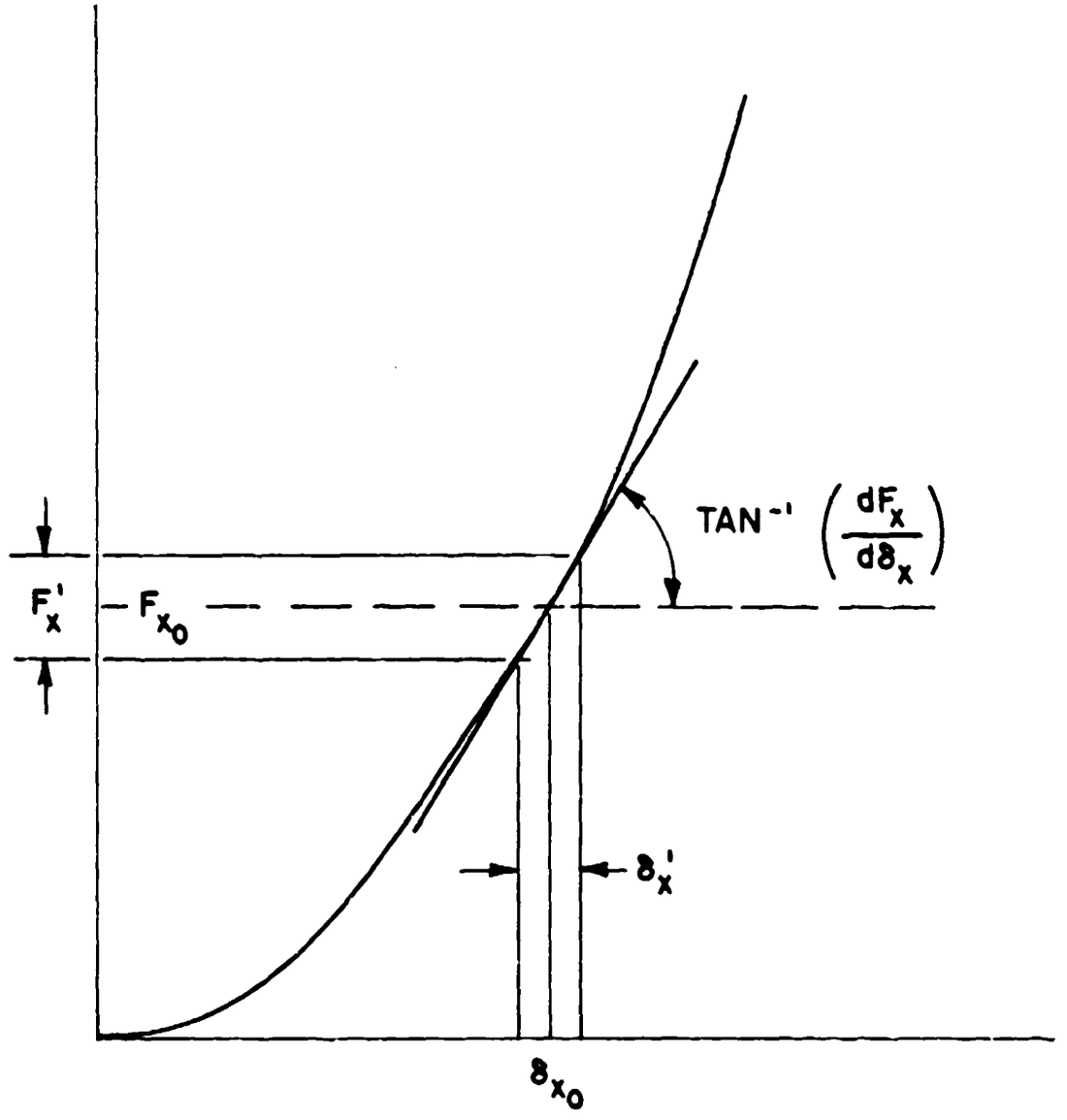


Figure 2 Linearization of Ball Bearing Stiffness

which are understood to be dynamic perturbation quantities unless the subscript "0" is used to designate the static equilibrium condition.

2.3 Ball Bearing Characterization

In many ways the ball bearing is much simpler to model from a rotor dynamics point of view than a fluid film bearing. In general, the following two simplifications can be made:

- The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- The restraining force/moment is normally temporally in phase with the displacement/inclination.

Figure 3 shows a ball bearing referred to in an orthogonal xyz coordinate system. The outer ring is fixed but the inner ring may move with respect to the coordinate system. Both rings are free to rotate about their axes.

Three lineal displacements, δ_x, δ_y, δ_z, and two angular displacements, θ_x, θ_y, are required to define the spatial position and attitude of the inner ring when it is displaced from its initial position. For purposes of derivation the initial situation is that existing when the bearing's end play is just taken up in the thrust direction. Figure 3 shows these displacements in the positive sense.

Figure 4 shows some important dimensions and establishes the convention of the ball-position index q. The contact angle β is the initial mounted contact angle and is shown in the positive sense.

2.3.1 Stiffness

The total characterization of a ball bearing's stiffness can be expressed by the matrix at the top of page 12.

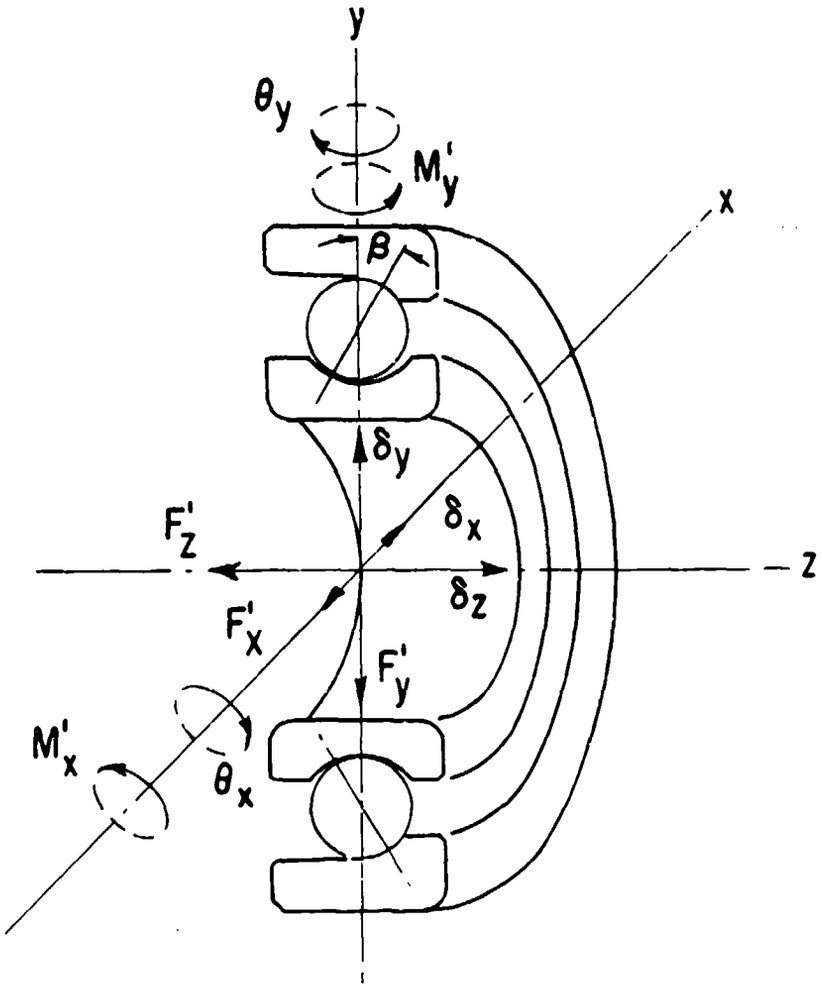


Figure 3 Ball Bearing Coordinate System

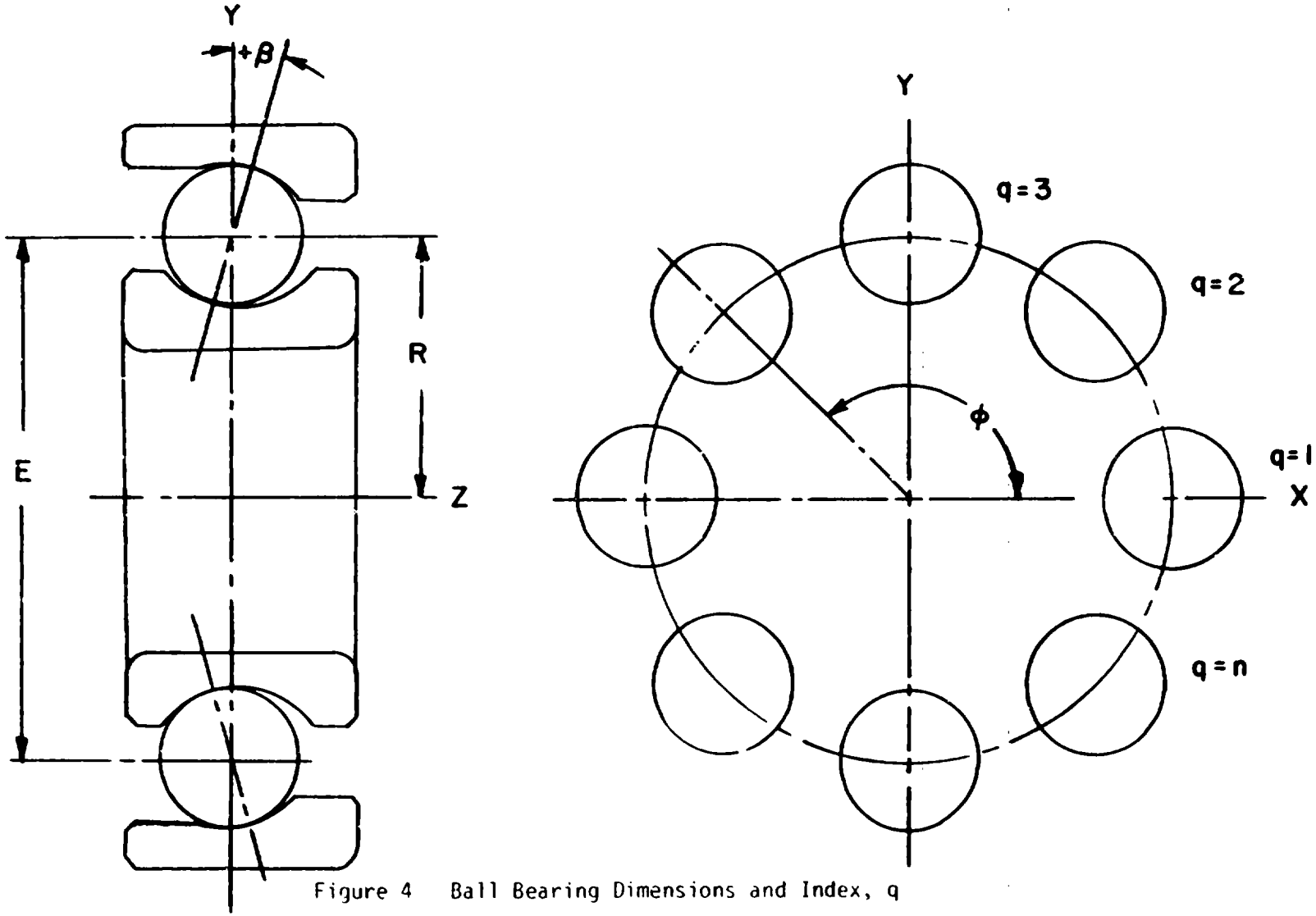


Figure 4 Ball Bearing Dimensions and Index, q

$$[K] = \begin{vmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_x}{\partial \theta_y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \theta_x} & \frac{\partial F_z}{\partial \theta_y} \\ \frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{vmatrix} \quad (13)$$

The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

$$(\underline{K})_{\text{lineal}} = \begin{vmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} \end{vmatrix} \quad (14)$$

$$(\underline{K})_{\text{angular}} = \begin{vmatrix} \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{vmatrix} \quad (15)$$

Note that although the axial components of stiffness are not utilized by the lateral rotor dynamics program (2), they have been retained in the general ball bearing stiffness matrix, Equation (13). The axial stiffness would be required, for example, if the reader was calculating the axial natural frequency of a ball bearing mounted shaft.

2.3.2 Damping

There is very little data on ball bearing damping. The data included in Reference 1, which is for nonrotating grease packed bearings, suggested a value in the order of 15-20 pound sec/in. This should be

used only as an approximate figure since this has not been confirmed.

2.4 Load-Deflection Relationships

For a given ball-raceway contact, the load deflection relationship is given by an equation of the form:

$$P = K \Delta^{3/2} \tag{16}$$

The total normal approach between two raceways under load separated by a rolling element is the sum of the approaches between the rolling element and each raceway. Hence,

$$\Delta = \Delta_1 + \Delta_2 \tag{17}$$

and

$$K = \left[\frac{1}{(1/K_1)^{2/3} + (1/K_2)^{2/3}} \right]^{3/2} \tag{18}$$

where K_1 and K_2 are a function of the ball-race geometry and material properties.

2.5 Ball Bearings Under Radial Load

For a rigidly supported bearing subjected to radial load, the radial deformation at any rolling element angular position is given by

$$\Delta_\phi = \delta_{max} \cos\phi - (1/2)P_D \tag{19}$$

in which δ_{max} is the maximum deflection, occurring at $\phi = 0$ and P_D is the diametral clearance. Figure 5 illustrates a radial bearing with clearance.

Equation (19) may be rearranged in terms of maximum deformation as follows:

$$\Delta = \Delta_{max} \{1 - \frac{1}{2\zeta} (1 - \cos\phi)\} \tag{20}$$

in which

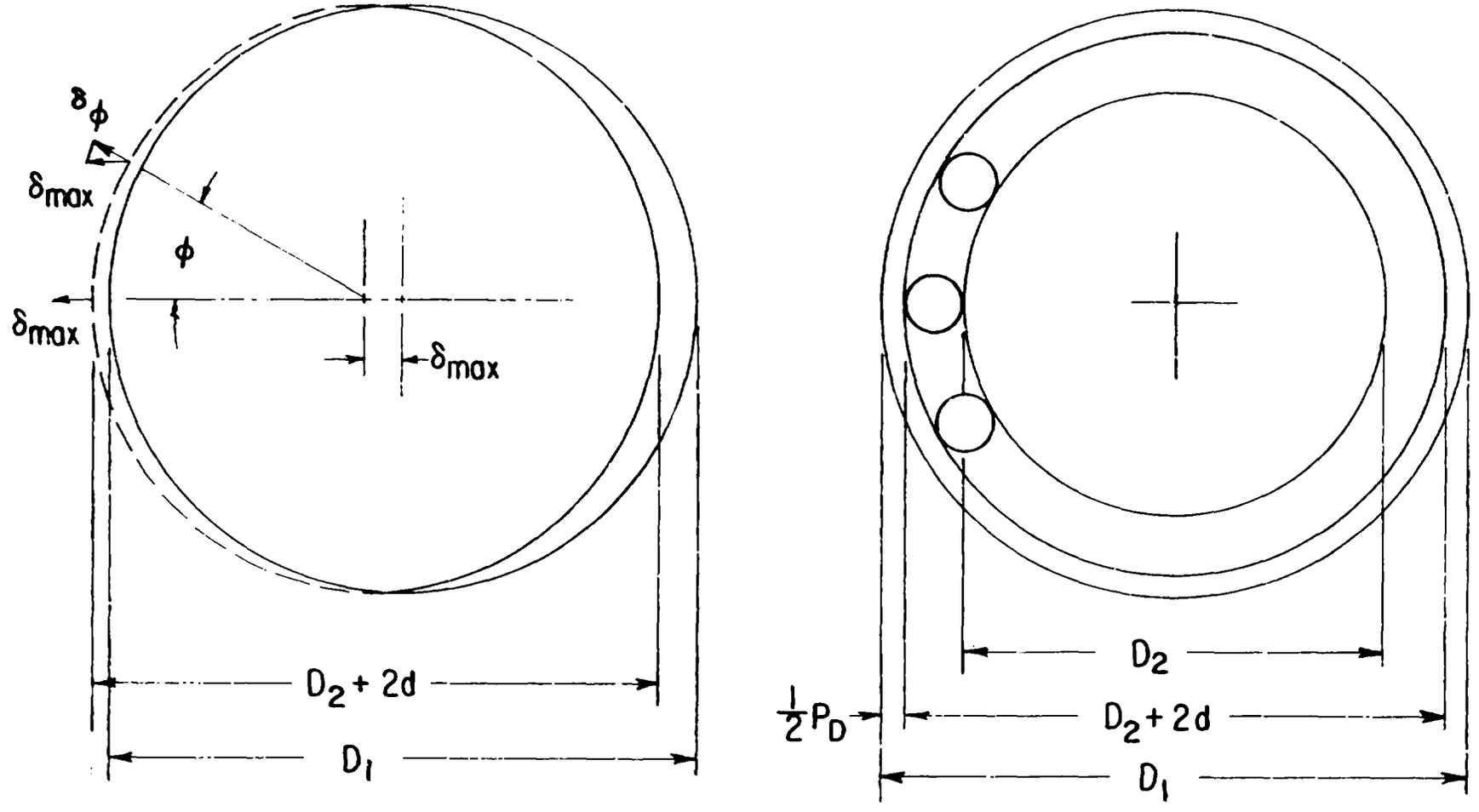


Figure 5 Radially Loaded Ball Bearing

$$\zeta = \frac{1}{2}\left[1 - \frac{P_D}{2\delta_{max}}\right] \tag{21}$$

It is clear from Equation (21) that the angular extent of the load zone is determined by the diametral clearance such that

$$\phi_\ell = \cos^{-1}\left[\frac{P_D}{2\delta_{max}}\right] \tag{22}$$

For zero clearance, $\phi = 90$

From Equation (16)

$$\left[\frac{P_\phi}{P_{max}}\right] = \left[\frac{\Delta_\phi}{\Delta_{max}}\right]^{3/2} \tag{23}$$

Therefore, from Equations (20) and (23)

$$P_\phi = P_{max}\left[1 - \frac{1}{2\zeta}(1 - \cos\phi)\right]^{3/2} \tag{24}$$

For static equilibrium to exist, the applied radial load must equal the sum of the horizontal (parallel to the load direction) components of the rolling element loads:

$$F_x = \sum_{\phi=-\pi}^{\phi=+\pi} P_\phi \cos\phi \tag{25}$$

$$F_x = P_{max} \sum_{\phi=-\pi}^{\phi=+\pi} \left[1 - \frac{1}{2\zeta}(1 - \cos\phi)\right]^{3/2} \cos\phi \tag{26}$$

The summation in above equations applies only to the angular extent of the load zone. Equation (26) can also be written in integral form:

$$F_x = nP_{max} \times \frac{1}{2\pi}\int_{-\phi_\ell}^{+\phi_\ell}\left[1 - \frac{1}{2\zeta}(1 - \cos\phi)\right]^{3/2}\cos\phi d\phi \tag{27}$$

or

$$F_x = nP_{max} J_x(\zeta) \tag{28}$$

in which

$$J_x(\zeta) = \frac{1}{2\pi} \int_{-\phi_\ell}^{+\phi_\ell} \{1 - \frac{1}{2\zeta}(1 - \cos\phi)\}^{3/2} \cos\phi d\phi \tag{29}$$

The radial integral of Equation (29) has been evaluated numerically by Harris (5) for various values of ζ.

From Equation (16),

$$P_{max} = K \Delta^{3/2}_{\phi=0} = K \{\delta_{max} - \frac{1}{2} P_D\}^{3/2} \tag{30}$$

Therefore,

$$F_x = nK \{\delta_{max} - \frac{1}{2} P_D\}^{3/2} J_x(\zeta) \tag{31}$$

For a given bearing with a given clearance under a given load, Equation (31) may be solved by trial and error.

For ball bearings under pure radial load and zero clearance, $J_x = .2285$, and it can be shown from Reference 6 that

$$P_{max} = \frac{4.37F_x}{n\cos\beta} \tag{32}$$

Accounting for nominal diametral clearance in the bearing, one may use the following approximation,

$$P_{max} = \frac{5F_x}{n\cos\beta} \tag{33}$$

2.5.1 Approximation to Radial Stiffness

A complete definition of bearing stiffness requires calculation of the stiffness matrix defined by Equation (13). However, for many

applications, simplifications can be made which permit calculation of radial stiffness of a ball bearing under radial load alone.

Palmgren (6) gives a series of formulas to calculate bearing deflection for specific conditions of loading. For slow and moderate speed deep-groove and angular-contact ball bearings subjected to radial load which causes only radial deflection, that is $\delta_z = 0$,

$$\delta_x = 1.58 \times 10^{-5} \frac{P_{max}^{2/3}}{d^{1/3}\cos\beta} \tag{34}$$

For self-aligning ball bearings,

$$\delta_x = 2.53 \times 10^{-5} \frac{P_{max}^{2/3}}{d^{1/3}\cos\beta} \tag{35}$$

For this case the maximum rolling element load has previously been shown to be

$$P_{max} = \frac{5F_x}{n\cos\beta} \tag{33}$$

Substituting for P_{max} in Equation (34) yields

$$\delta_x = \frac{4.62 \times 10^{-5} F_x^{2/3}}{n^{2/3} d^{1/3} \cos^{5/3}\beta} \tag{36}$$

Transposing, then taking the derivative with respect to x, the stiffness of the bearing is

$$K_{xx} = 4.77 \times 10^{6} n d^{1/2} \cos^{5/2}\beta \delta_x^{1/2} \tag{37}$$

For self-aligning ball bearings

$$K_{xx} = 2.36 \times 10^{6} n d^{1/2} \cos^{5/2}\beta \delta_x^{1/2} \tag{38}$$

From Equations (37) and (38) it is apparent that the deflection-stiffness relationship is nonlinear because it is dependent on the square

root of radial deflection. In this respect, a ball bearing is unlike a simple spring for which deflection is linear with respect to load.

By substituting for δ_x in Equations (37) and (38), bearing stiffness can be expressed in terms of load as:

Angular Contact Bearings

$$\frac{K_{xx}}{d^{1/3} n^{2/3} \cos^{5/3}\beta} = 3.247 \times 10^4 \, F_x^{1/3} \tag{39}$$

Self-Aligning Bearings

$$\frac{K_{xx}}{d^{1/3} n^{2/3} \cos^{5/3}\beta} = 2.028 \times 10^4 \, F_x^{1/3} \tag{40}$$

These expressions are plotted in Figure 6.

2.6 Bearings Under Thrust Load

Thrust ball bearings subjected to a centric thrust load have the load distributed equally among the rolling elements. Hence,

$$P = \frac{F_z}{n \sin\beta} \tag{41}$$

In Equation (41), β is the contact angle which occurs in the loaded bearing. For thrust ball bearings whose contact angles are nominally less than 90 degrees, the contact angle in the loaded bearing is greater than the initial contact angle β' which occurs in the unloaded bearing.

In the absence of centrifugal loading, the contact angles at inner and outer raceways are identical; however, they are greater than those in the unloaded condition. In the unloaded condition, contact angle is defined by

$$\cos\beta' = 1 - \frac{P_D}{2Bd} \tag{42}$$

in which P_D is the mounted diametral clearance. A thrust load, F_z, applied to the inner ring as shown by Figure 7 causes an axial deflection δ_z. This axial deflection is a component of a normal deflection along the line of contact such that from Figure 7

$$\Delta = Bd \left(\frac{\cos\beta'}{\cos\beta} - 1\right) \tag{43}$$

Since $P = K\Delta^{3/2}$,

$$P = K(Bd)^{3/2} \left(\frac{\cos\beta'}{\cos\beta} - 1\right)^{3/2} \tag{44}$$

Substitution of Equation (41) into Equation (44) yields

$$\frac{F_z}{nK(Bd)^{3/2}} = \sin\beta \left(\frac{\cos\beta'}{\cos\beta} - 1\right)^{3/2} \tag{45}$$

Since K is a function of the final contact angle, β, Equation (45) must be solved by trial and error to yield an exact solution for β. Jones (7) has defined an axial deflection constant K_z as follows:

$$K_z = \frac{B}{g(+\gamma) + g(-\gamma)} \tag{46}$$

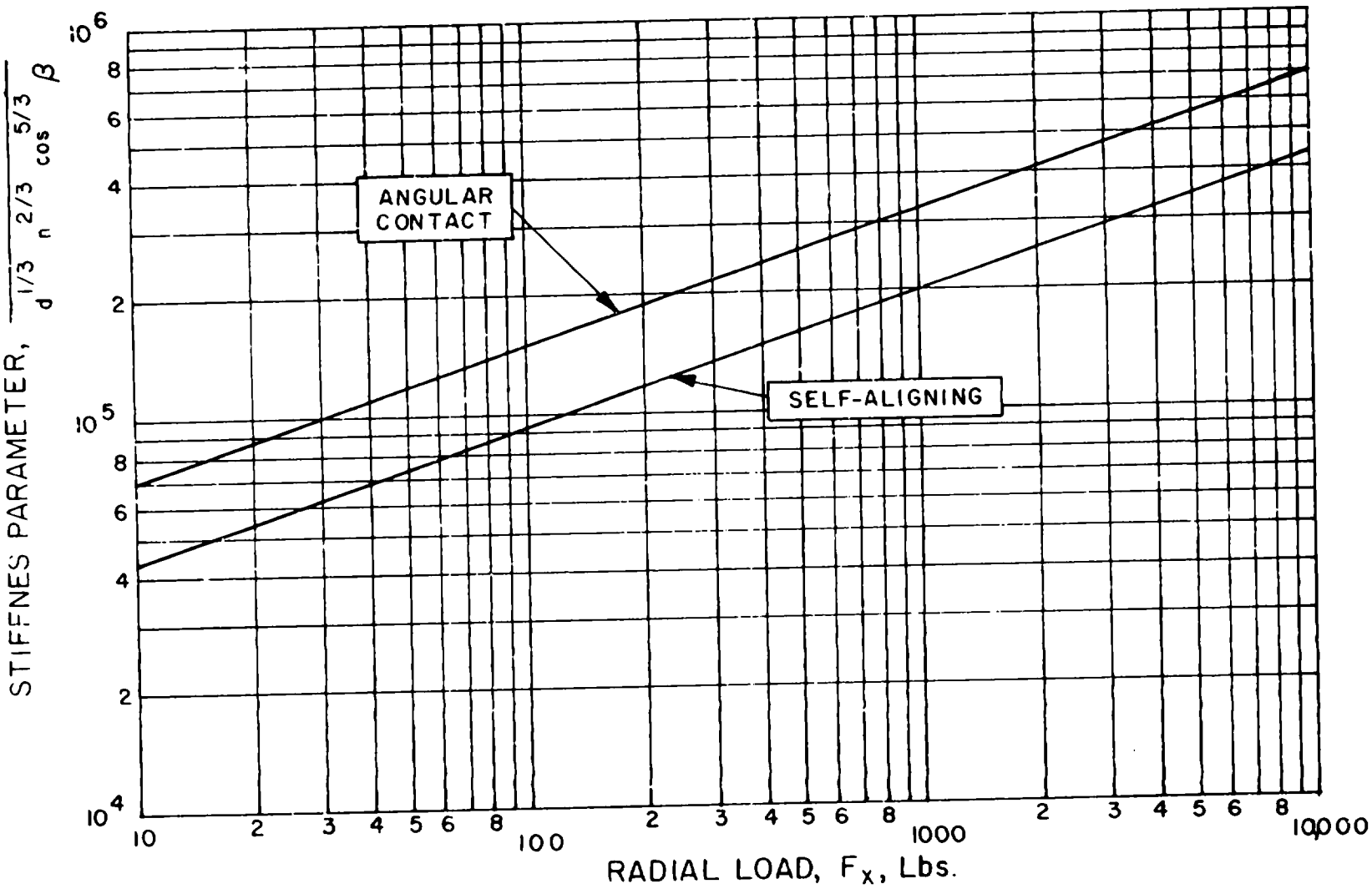
in which $\gamma = \frac{d \cos \beta}{E}$ and $g(+\gamma)$ refers to the inner raceway and $g(-\gamma)$ refers to the outer raceway. Jones further indicates that the sum of $g(+\gamma)$ and $g(-\gamma)$ remains virtually constant for all contact angles being dependent only on total curvature B. The axial deflection constant K_z is related to K as follows:

$$K = \frac{K_z d^{1/2}}{B^{3/2}} \tag{47}$$

Hence:

$$\frac{F_z}{nd^2K_z} = \sin\beta \left(\frac{\cos\beta'}{\cos\beta} - 1\right)^{3/2} \tag{48}$$

Taking K_z from Reference 7, Equation (48) may be solved numerically by the



RADIAL STIFFNESS PARAMETER
VS.
RADIAL LOAD

Figure 6 Radial Stiffness Parameter vs. Radial Load

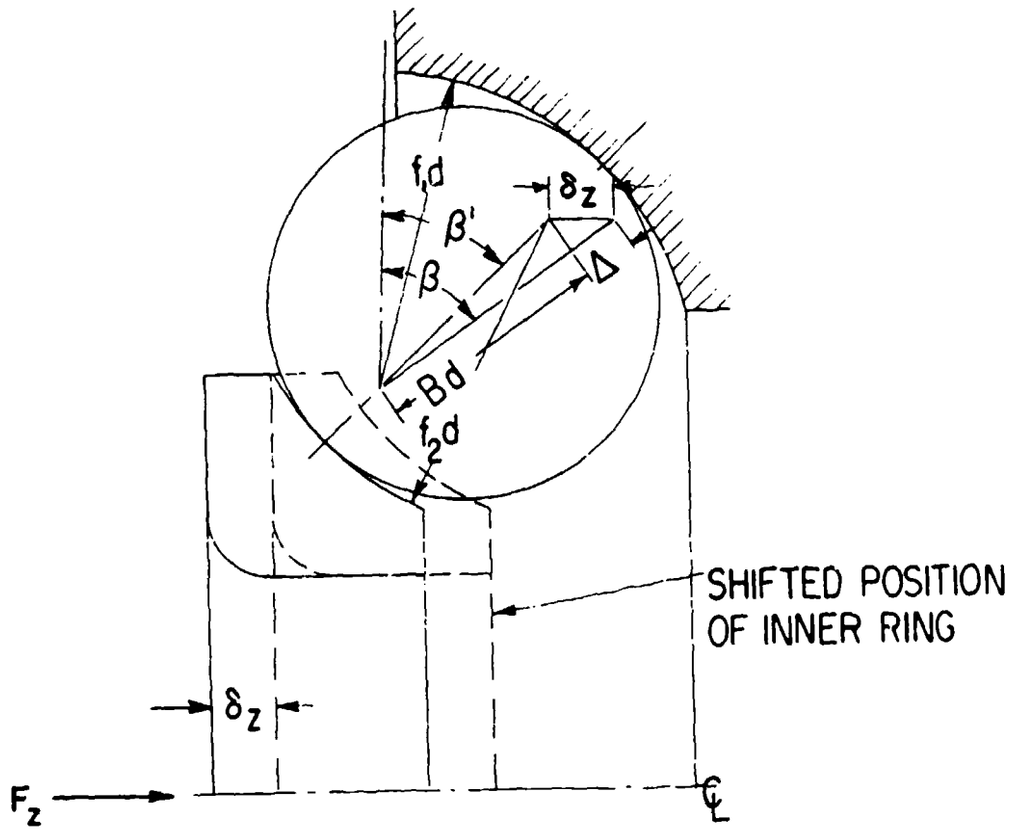


Figure 7 Angular Contact Ball Bearing Under Thrust Load

Newton-Raphson method.

The axial deflection δ_z corresponding to Δ may also be determined from Figure 7 as follows:

$$\delta_z = (Bd + \Delta) \sin\beta - Bd \sin\beta' \tag{49}$$

Substituting Δ from Equation (43) yields

$$\delta_z = \frac{Bd \sin(\beta-\beta')}{\cos\beta} \tag{50}$$

2.6.1 Approximation to Axial Stiffness

From examination of Figure 7, the following approximation can be made:

$$\Delta \sim \delta_z \sin\beta \tag{51}$$

Substituting for Δ in Equation (16) gives

$$\delta_z \sim \frac{1}{K^{2/3}} \frac{P^{2/3}}{\sin\beta} \tag{52}$$

Using the Jones (7) axial deflection parameter, Equation (52) becomes

$$\delta_z \sim \frac{B}{K_z^{2/3} d^{1/3}} \frac{P_{max}^{2/3}}{\sin\beta} \tag{53}$$

A similar approximation was arrived at by Palmgren (6). Suggested values for axial deflection under pure axial load are:

Angular-Contact Bearings

$$\delta_z = 1.58 \times 10^{-5} \frac{P_{max}^{2/3}}{d^{1/3} \sin\beta} \tag{54}$$

Self-Aligning Ball Bearings

$$\delta_z = 2.53 \times 10^{-5} \frac{P_{max}^{2/3}}{d^{1/3} \sin\beta} \tag{55}$$

Thrust Ball Bearings

$$\delta_z = 1.9 \times 10^{-5} \frac{p_{max}^{2/3}}{d^{1/3} \sin\beta} \tag{56}$$

The corresponding stiffnesses are:

Angular-Contact Bearings

$$\frac{K_{zz}}{d^{1/3} n^{2/3} \sin^{5/3}\beta} = 9.5 \times 10^4 \, F_z^{1/3} \tag{57}$$

Self-Aligning Ball Bearings

$$\frac{K_{zz}}{d^{1/3} n^{2/3} \sin^{5/3}\beta} = 5.9 \times 10^4 \, F_z^{1/3} \tag{58}$$

Thrust Ball Bearings

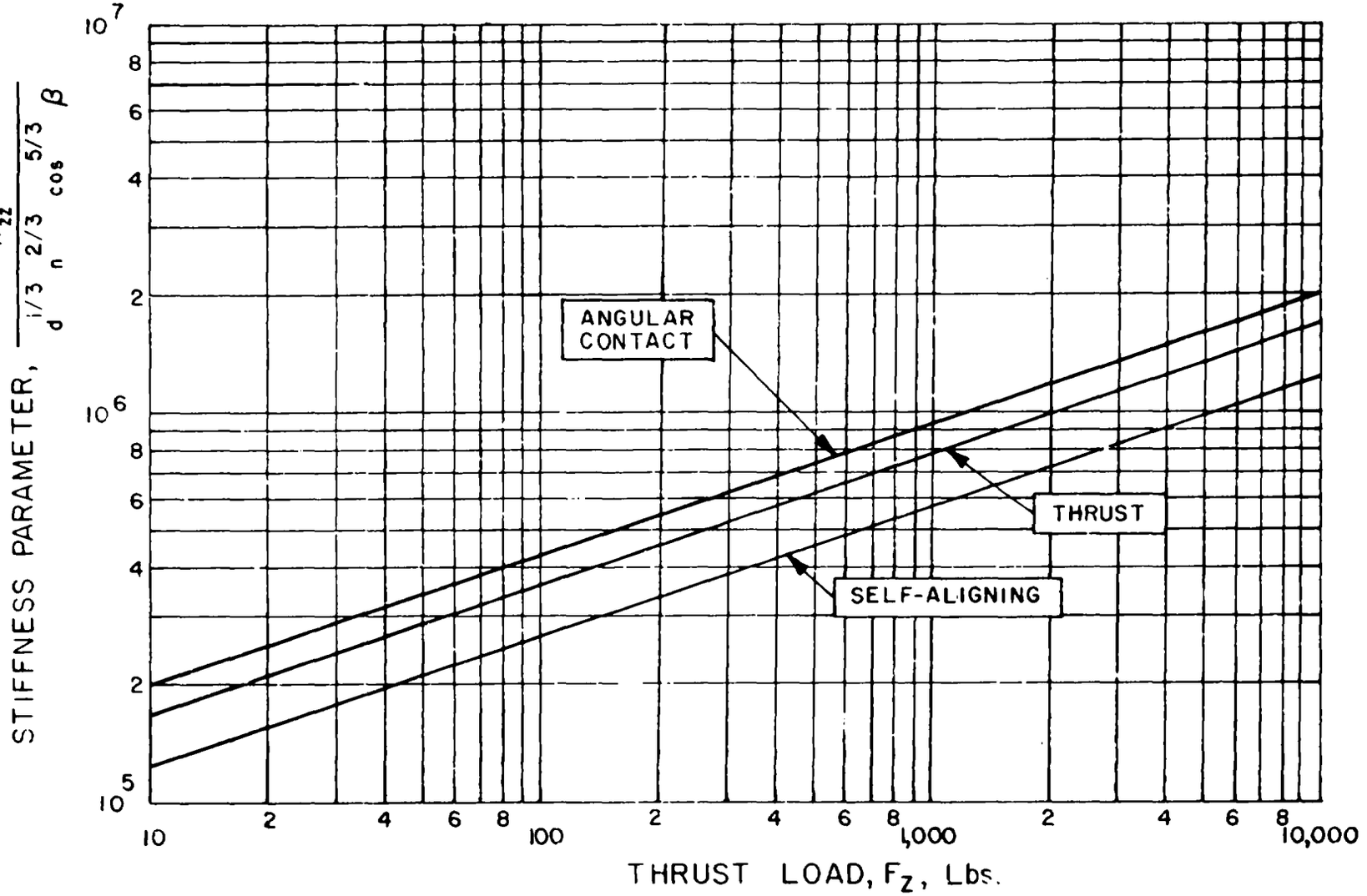
$$\frac{K_{zz}}{d^{1/3} n^{2/3} \sin^{5/3}\beta} = 7.9 \times 10^4 \, F_z^{1/3} \tag{59}$$

These relationships are plotted in Figure 8 for the three types of bearings.

2.7 Ball Bearings Under Combined Loading

Except for ball bearings under simple radial or thrust load, treated previously, there are very few solutions for bearing stiffness that can be evaluated by simple hand computation.

When a ball bearing operates at high speed, the body forces resulting from the ball's motion become significant and must be considered in any analysis. Figure 9 shows the forces and moments acting on the j^{th} ball in a high-speed ball bearing. The operating contact angle at the outer contact is less than that on the inner because of the body forces. Subscript 1 refers to an outer contact and subscript 2 to an inner.



THRUST STIFFNESS PARAMETER
VS.
THRUST LOAD

Figure 8 Thrust Stiffness Parameter vs. Thrust Load

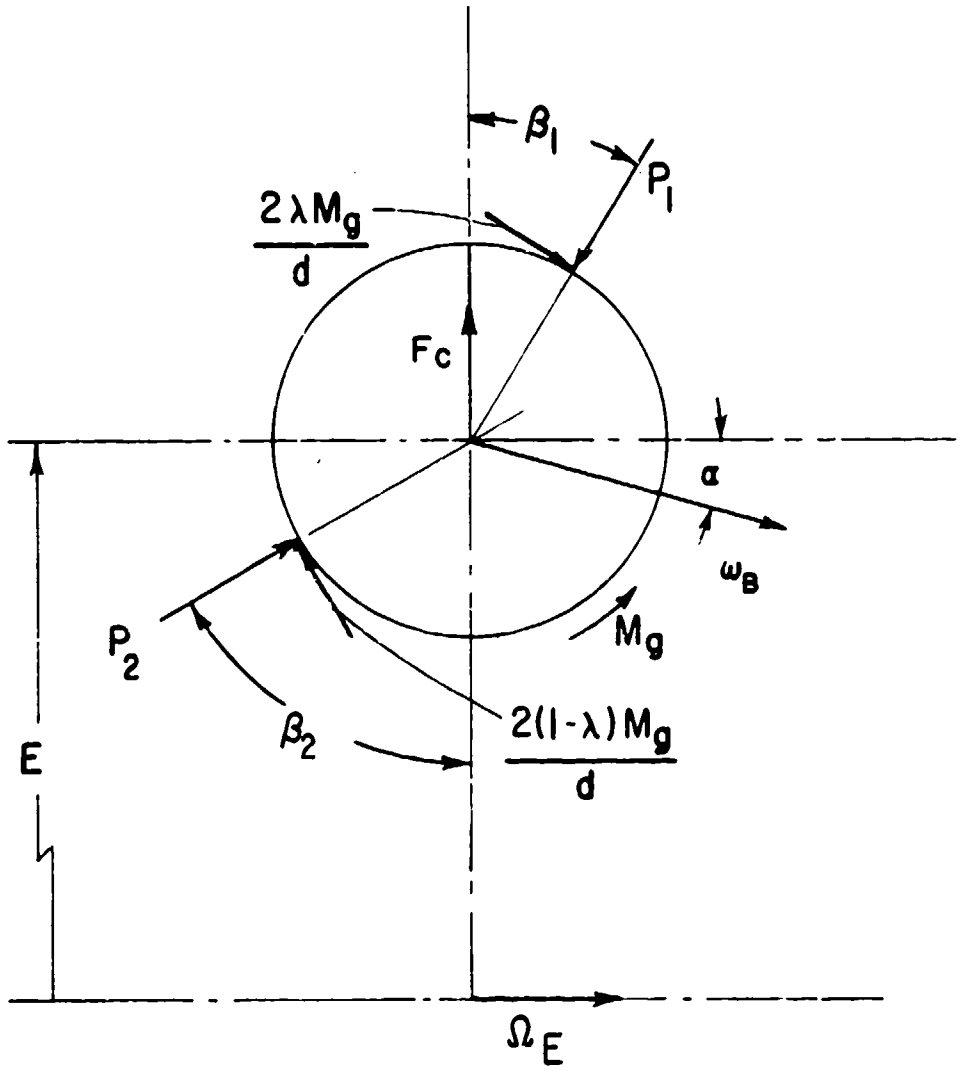


Figure 9 Ball Forces at High Speed Conditions

In Figure 10 the ball center is fixed in the plane of the paper. The ball rotates with the angular velocity ω_B directed at the angle α. race rotates about the bearing axis with the angular velocity ω_i, i ,2 relative to the retainer. For the linear velocity of race and ball to be equal at a contact, the following relation must be satisfied:

$$\omega_B = \frac{C_i E'(1 + C_i \gamma_i)\omega_i}{d\cos(\beta_i - \alpha)} \tag{60}$$

where $C_1 = 1$ and $C_2 = -1$

$$\gamma_i = \frac{d\cos\beta_i}{E'} \tag{61}$$

For the i^{th} race to be stationary, the ball must orbit with the angular velocity Ω_E such that

$$\Omega_E = -\omega_i \tag{62}$$

With stationary outer race and rotating inner, the actual angular velocity of the inner race is

$$\Omega_2 = \omega_2 + \Omega_E = \omega_2 - \omega_1 \tag{63}$$

For rotating outer and stationary inner, the actual angular velocity of the outer is

$$\Omega_1 = \omega_1 + \Omega_E = \omega_1 - \omega_2 \tag{64}$$

From Equation (60),

$$\frac{\omega_1}{\omega_2} = -\frac{(1-\gamma_2)\cos(\beta_1 - \alpha)}{(1+\gamma_1)\cos(\beta_2 - \alpha)} \tag{65}$$

considering that both races may rotate there results:

$$\omega_1 = \frac{(\Omega_1 - \Omega_2)(1-\gamma_2)\cos(\beta_1 - \alpha)}{(1+\gamma_1)\cos(\beta_2 - \alpha) + (1-\gamma_2)\cos(\beta_1 - \alpha)} \tag{66}$$

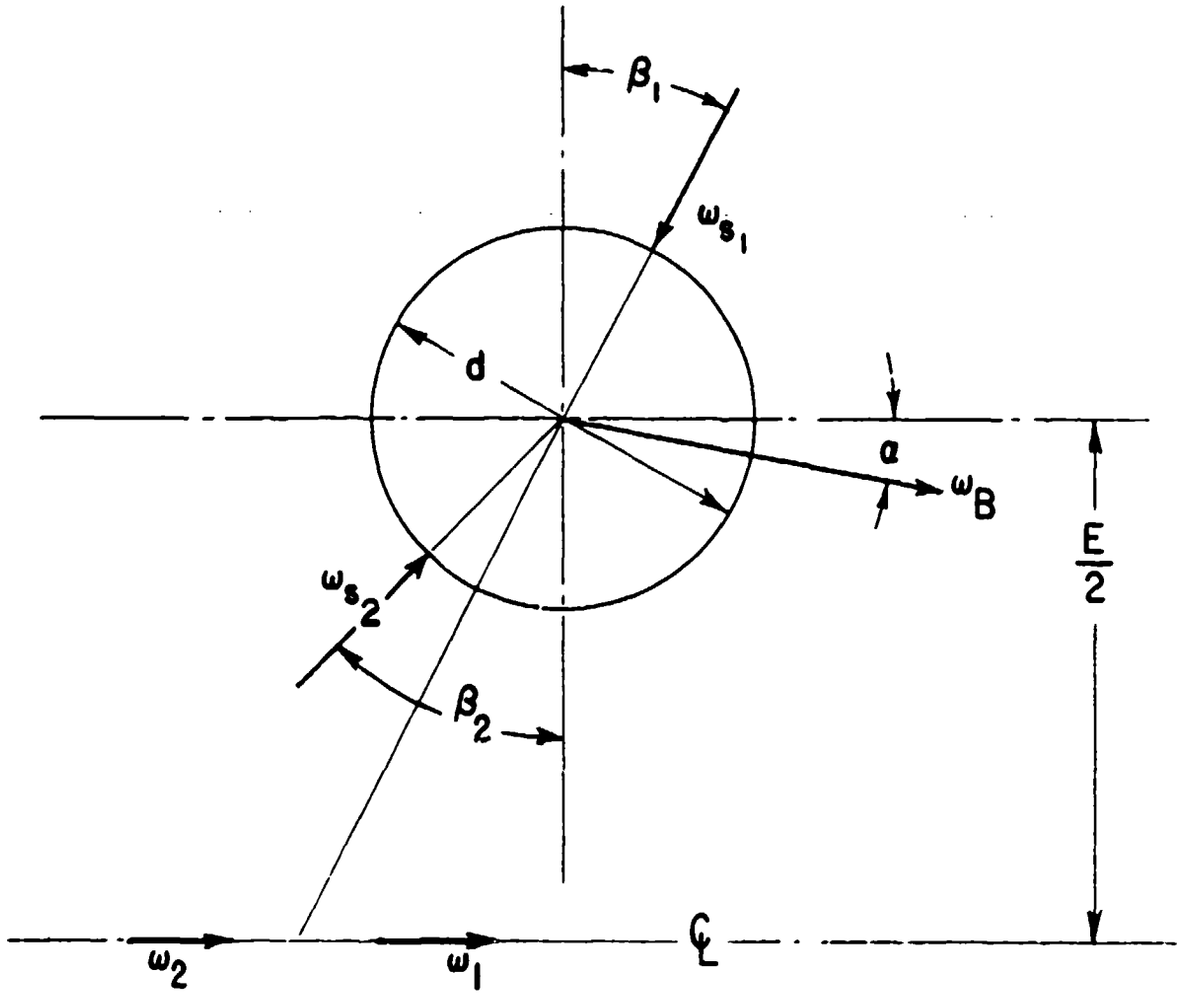


Figure 10 Ball Motion Vectors

$$\omega_2 = -\frac{(\Omega_1-\Omega_2)(1+\gamma_1)\cos(\beta_2-\alpha)}{(1+\gamma_1)\cos(\beta_2-\alpha)+(1-\gamma_2)\cos(\beta_1-\alpha)} \tag{67}$$

$$\omega_B = \frac{E'\{(\Omega_1-\Omega_2)(1+\gamma_1)(1-\gamma_2)\}}{d\{(1+\gamma_1)\cos(\beta_2-\alpha)+(1-\gamma_2)\cos(\beta_1-\alpha)\}} \tag{68}$$

$$\Omega_E = \frac{\Omega_1(1+\gamma_1)\cos(\beta_2-\alpha)+\Omega_2(1-\gamma_2)\cos(\beta_1-\alpha)}{(1+\gamma_1)\cos(\beta_2-\alpha)+(1-\gamma_2)\cos(\beta_1-\alpha)} \tag{69}$$

For an arbitrary choice of α there will be a spin of the ball relative to a race about the normal at the center of the contact area. This is illustrated in Figure 11.

From Figure 10,

$$\omega_{s_i} = C_i\{-\omega_i\sin\beta_i + \omega_B\sin(\beta_i-\alpha)\} \tag{70}$$

The controlling race hypothesis assumes that all spin occurs at one contact while no spin occurs at the other. The contact at which no spin exists is called the controlling race. Lightly-loaded bearings may depart somewhat from this situation.

If ω_{s_i} is made zero and Equation (70) solved for α, there results for outer race control:

$$\alpha = \tan^{-1}\frac{\sin\beta_1\cos\beta_1}{\cos^2\beta_1+\gamma_1} \tag{71}$$

and for inner race control:

$$\alpha = \tan^{-1}\frac{\sin\beta_2\cos\beta_2}{\cos^2\beta_2-\gamma_2} \tag{72}$$

The existence of a particular type of control depends on the relative torques required to produce spin at the two contacts.

The torque required to produce spin is:

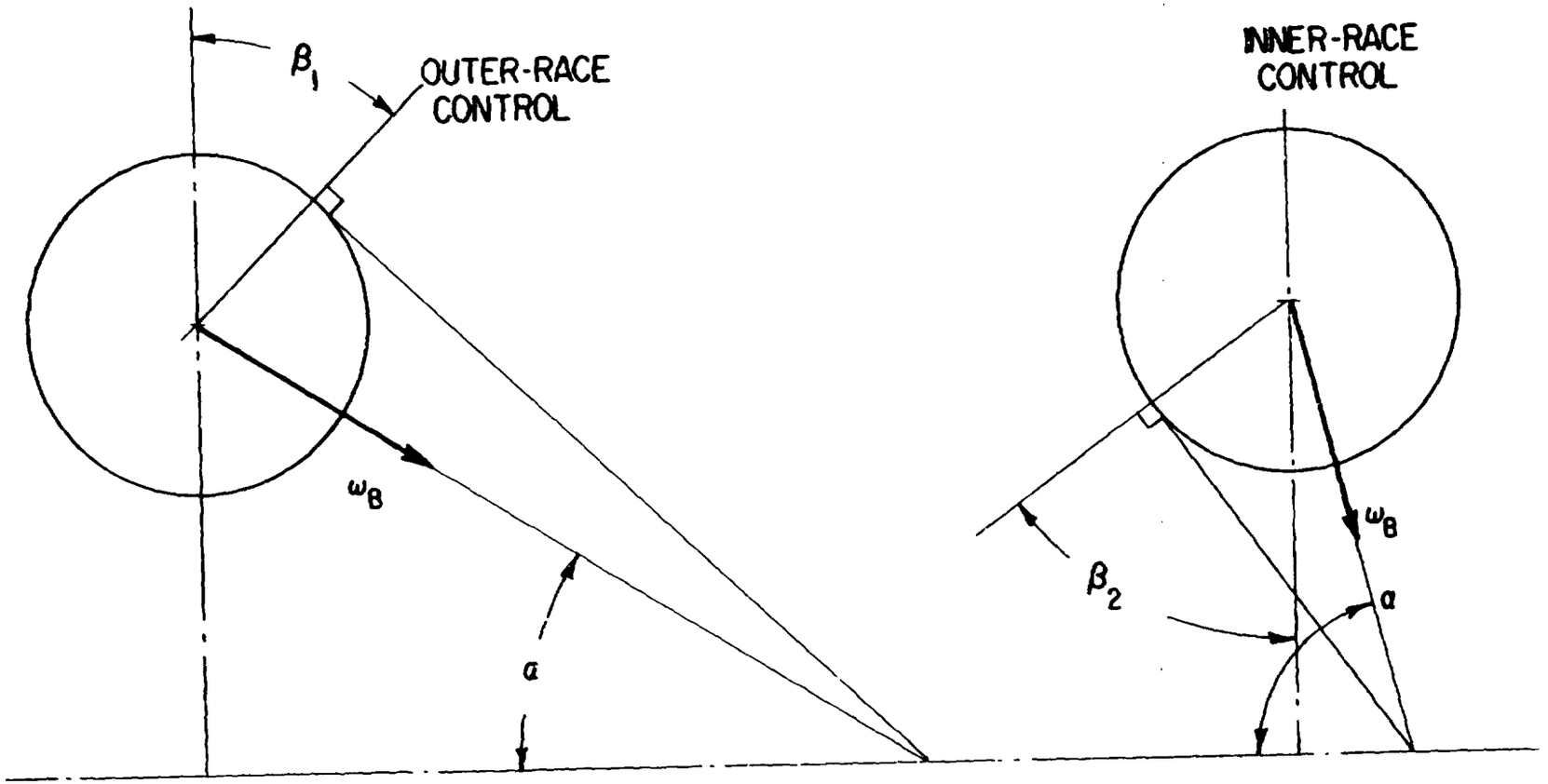


Figure 11 Illustration of Race Control Concept

$$Q = \frac{3\mu P\, a\, E(\varepsilon)}{8} \quad . \tag{73}$$

Q = spin torque, lb-in

P = contact load, lbs.

μ = coefficient of sliding friction

a = semi-major axis of pressure ellipse, in.

$E(\varepsilon)$ = complete elliptic integral of the second kind formed with the modulus sinε where cosε = b/a, b being the semi-minor axis of the ellipse

Figure 12 shows a ball acted on by the spin torque vectors Q_1 and Q_2. Outer race control will exist if Q_1 projected on Q_2 is greater than Q_2. Inner race control will exist if Q_2 projected on Q_1 is greater than Q_1.

Figure 13 shows the relative positions of ball and race curvature centers before and after the application of the five displacements, δ_x, δ_y, δ_z, θ_x, and θ_y. The outer race curvature center is fixed. The inner race curvature center has moved to (A_1, A_2) and the ball center to (X_1, X_2).

$$A_1 = Bd\sin\beta + \delta_z + \delta_z'' + R[(\theta_x + \theta_x'')\sin\phi + (\theta_y + \theta_y'')\cos\phi] \tag{74}$$

$$A_2 = Bd\cos\beta + (\delta_x + \delta_x'')\cos\phi + (\delta_y + \delta_y'')\sin\phi - \frac{P_D}{2} \tag{75}$$

$$B = f_1 + f_2 - 1 \tag{76}$$

$$R = E/2 + (f_2 - .5)\, d\cos\beta \tag{77}$$

P_D = diametral clearance, in.

δ_x'' = initial displacement along x, in.

δ_y'' = initial displacement along y, in.

δ_z'' = initial displacement along z, in.

θ_x'' = initial misalignment about x, rad.

θ_y'' = initial misalignment about y, rad.

For the forces and moments acting on the ball to be in equilibrium, the following must be satisfied:

$$P_1\sin\beta_1 - P_2\sin\beta_2 - \frac{2M}{d}\{\lambda\cos\beta_1 - (1-\lambda)\cos\beta_2\} = 0 \tag{78}$$

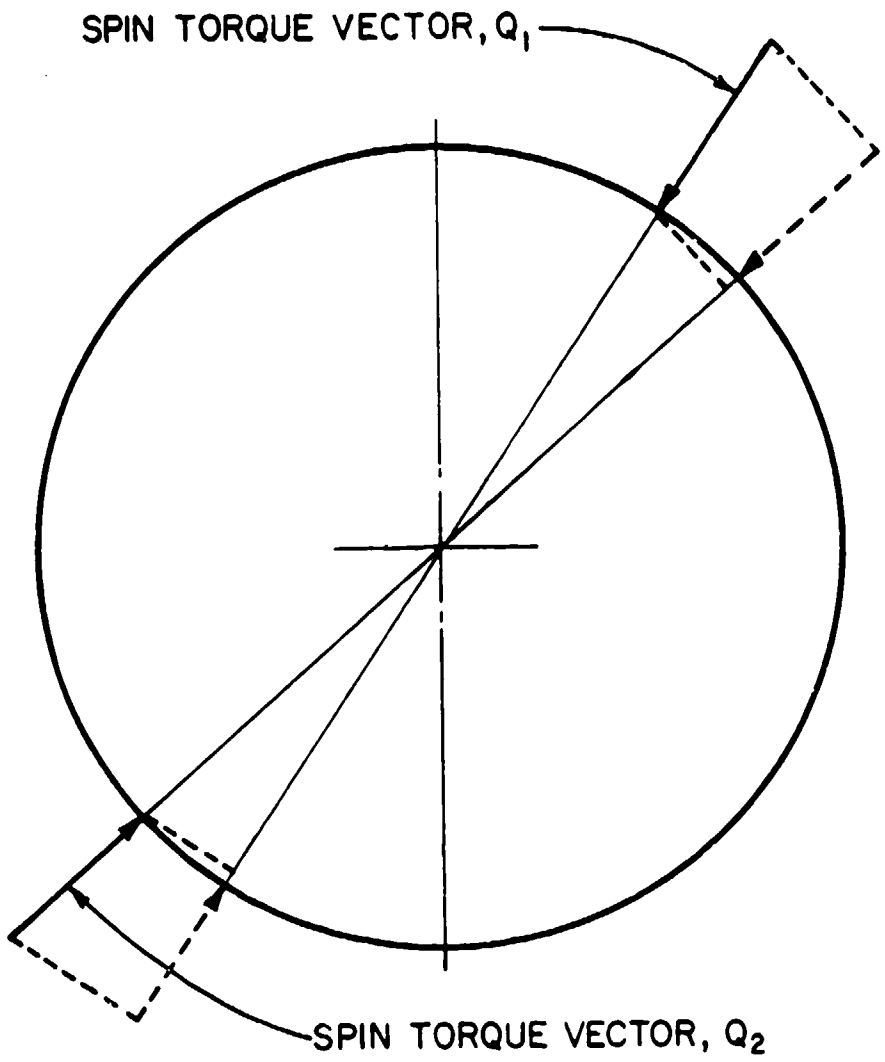


Figure 12 Spin Torque Vectors

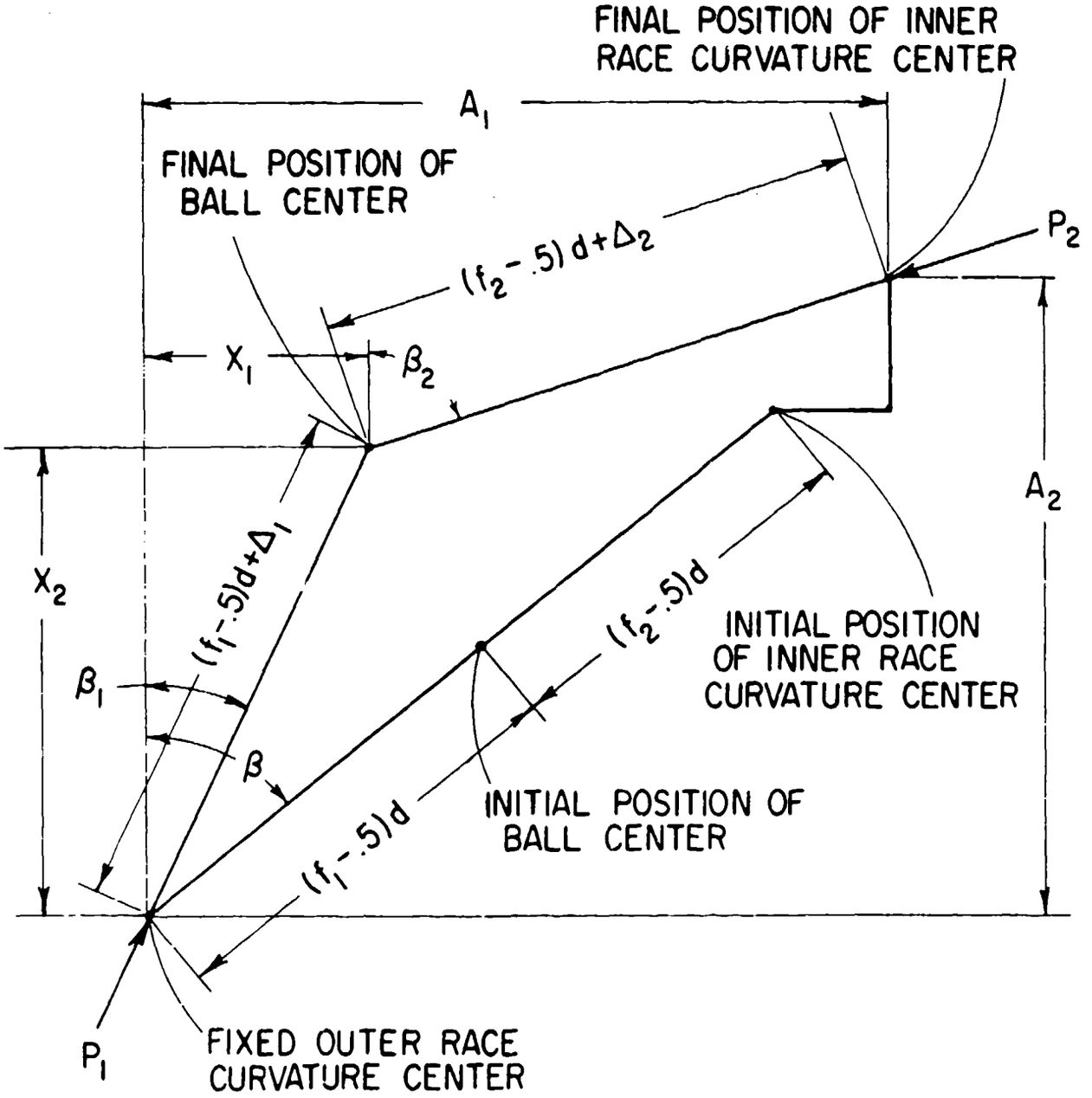


Figure 13 Race Curvature Center Deflection

$$P_1\cos\beta_1 - P_2\cos\beta_2 + \frac{2M}{d}\{\lambda\sin\beta_1-(1-\lambda)\sin\beta_2\} - F_c = 0 \tag{79}$$

where λ is 1 for outer race control and 0 for inner race control and

$$F_c = M\,\Omega_E^2\,E'/2 \tag{80}$$

$$M = I_P\,\Omega_E\,\omega_B\sin\alpha \tag{81}$$

E' = Operating Pitch Diameter, in.

$$= E + 2X_2 - (f_1 - .5)d\cos\beta \tag{82}$$

From Figure 13

$$\beta_1 = \tan^{-1}\frac{X_1}{X_2} \tag{83}$$

$$\beta_2 = \tan^{-1}\frac{A_1-X_1}{A_2-X_2} \tag{84}$$

Δ_1 and Δ_2 are the elastic approaches of ball and raceways at the contacts

$$\Delta_1 = (X_1^2 + X_2^2)^{1/2} - (f_1 - .5)d > 0 \tag{85}$$

$$\Delta_2 = \{(A_1 - X_1)^2 + (A_2 - X_2)^2\}^{1/2} - (f_2 - .5)d > 0 \tag{86}$$

The contact loads are related to the elastic approaches through

$$P_i = K_i\,\Delta_i^{3/2} \tag{87}$$

where

$$K_i = \frac{11.84771}{(\eta_R + \eta_B)\cos\varepsilon_i}\left[\frac{E(\varepsilon_i)d}{\{K(\varepsilon_i)\}^3\{4 - 1/f_i - \frac{2C_i\gamma_i}{1+C_i\gamma_i}\}}\right]^{1/2} \tag{88}$$

$$\eta_R = \frac{4(1-\nu_R^2)}{E_R} \tag{89}$$

$$n_B = \frac{4(1-\nu_B^2)}{E_B} \tag{90}$$

E_R and E_B are moduli of elasticity for race and ball.
ν_R and ν_B are Poisson's Ratios for race and ball.
$K(\varepsilon)$ and $E(\varepsilon)$ are the complete elliptic integrals of the first and second kind having the modulus $\sin\varepsilon$.

$$\sin\varepsilon_i = \sqrt{1 - \frac{b_i^2}{a_i^2}} \tag{91}$$

b_i and a_i are the semi-minor and semi-major axes of the pressure ellipse
ε is related to the contact geometry through

$$\cot^2\varepsilon_i = \frac{(1-\cos\tau_i)E(\varepsilon_i)}{2[K(\varepsilon_i) - E(\varepsilon_i)]} \tag{92}$$

$$\cos\tau_i = \frac{\frac{1}{f_i} - \frac{2C_i\gamma_i}{1+C_i\gamma_i}}{4 - \frac{1}{f_i} - \frac{2C_i\gamma_i}{1+C_i\gamma_i}} \tag{93}$$

Equations (78) and (79) are a set of nonlinear, simultaneous equations. When the relative ring displacements are held constant, the variables in Equations (78) and (79) are X_1 and X_2. These are evaluated numerically.

The reactions of the bearing on the shaft are

$$F'_x = \sum_{i=1}^{n} \left[P_{2_i}\cos\beta_{2_i} + \frac{2(1-\lambda_i)M_{g_i}}{d}\sin\beta_{2_i}\right]\cos\phi_i \tag{94}$$

$$F'_y = \sum_{i=1}^{n} \left[P_{2_i}\cos\beta_{2_i} + \frac{2(1-\lambda_i)M_{g_i}}{d}\sin\beta_{2_i}\right]\sin\phi_i \tag{95}$$

$$F'_z = \sum_{i=1}^{n} \left[P_{2_i}\sin\beta_{2_i} - \frac{2(1-\lambda_i)M_{g_i}}{d}\cos\beta_{2_i}\right] \tag{96}$$

$$M'_x = \sum_{i=1}^{n} \left[RP_{2_i} \sin\beta_{2_i} - \frac{2(1-\lambda_i)M_{g_i}}{d} (R\cos\beta_{2_i} - f_2 d)\right]\sin\phi_i \qquad (97)$$

$$M'_y = \sum_{i=1}^{n} \left[RP_{2_i} \sin\beta_{2_i} - \frac{2(1-\lambda_i)M_{g_i}}{d} (R\cos\beta_{2_i} - f_2 d)\right]\cos\phi_i \qquad (98)$$

Then the equilibrium of external forces

$$\sum_{i=x,y,z} \{F'_i + F_i\} = 0 \qquad (99)$$

F_x = external load along x, lbs.
F_y = external load along y, lbs.
F_z = external load along z, lbs.

Equation (99) is a set of three non-linear simultaneous equations in which the variables are δ_x, δ_y and δ_z. They are solved numerically in Appendix A using Newton-Raphson iteration.

If $\delta_{i_{m-1}}$ are current estimates of the three displacements, improved estimates are

$$\delta_{i_m} = \delta_{i_{m-1}} - \left[K_{ij}\right]^{-1} \left\{\Gamma_j\right\}_{m-1} \qquad (100)$$

where $\left\{\Gamma_j\right\}_{m-1}$ are the residues of Equation (99) evaluated at current estimates of δ_i.

The elements K_{ij} of the coefficient matrix are the partial derivatives of the bearing's reactions with respect to the displacements δ_j and comprise the stiffness matrix, Equation (13).

The elements K_{ij} of the stiffness matrix, Equation (13), form the data input for the rotordynamic response computer program described in Reference 2.

SECTION III

APPLICATION OF COMPUTER PROGRAM

The analysis of Section 2.0 has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in Appendix A.

3.1 Input Format

Instructions for preparation of the input cards are included as comment cards in the program listing in Appendix A. To illustrate a typical case consider the bearing in Table 1.

TABLE 1

SAMPLE CASE
DEEP GROOVE BALL BEARING

Number of Balls	=	9
Ball Diameter	=	.1875 inches
Pitch Diameter	=	.9252 inches
Contact Angle	=	0^{o}
Outer-race Curvature	=	.530
Inner-race Curvature	=	.516
Poisson's Ratio	=	.25
Modulus of Elasticity	=	29×10^{6} lb/in.
Ball Density	=	0.283 lb/in.^{3}
RPM of Inner Race	=	1000 rpm
RPM of Outer Race	=	0 rpm
Radial Load	=	1000 lbs.
Axial Load	=	0 lbs.

Figure 14 shows a printout of input for this case.

TEST CASE
TABLE 1, PAGE 36 OF AFAPL-TR-77, PART II

DESIGN DATA FOR BEARING NO. 1

NO. OF BALLS	BALL DIAMETER IN	PITCH DIAMETER IN	INITIAL CONTACT ANGLE DESIGN DEG	INITIAL CONTACT ANGLE MOUNTED DEG	RACE CURVATURE OUTER	RACE CURVATURE INNER	INTERNAL CLEARANCE INCREMENT IN	INTERNAL CLEARANCE OPERATING IN	MATERIAL DENSITY LB/IN**3
9.0000+00	1.875_-01	9.2526-01	0.0000	0.0000	5.3006-01	5.1600-01	0.0000	0.0000	2.9300-01

MODULUS OF ELASTICITY OUTER LB/IN**2	MODULUS OF ELASTICITY INNER LB/IN**2	MODULUS OF ELASTICITY BALLS LB/IN**2	POISSON'S RATIO OUTER	POISSON'S RATIO INNER	POISSON'S RATIO BALLS
2.9000+07	2.9000+07	2.9000+07	2.5000-01	2.5000-01	2.5000-01

INPUT DATA FOR RUN NO. 1 BEARING NO. 1

RPM OF OUTER	RPM OF INNER	LOADS APPLIED TO SHAFT ALONG X LB	LOADS APPLIED TO SHAFT ALONG Z LB	INITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER ALONG X IN	ALONG Y IN	ALONG Z IN	ABOUT X RADIANS	ABOUT Y RADIANS
0.0000	1.0000+03	1.0000+03	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

1. = DEFLECTION PERMITTED

ALONG X	ALONG Y	ALONG Z
1.0000+00	0.0000	0.0000

Figure 14 Input Data for Sample Problem

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3.2 Output Format

Figure 15 shows the output format for the sample case of Table 1. The output data includes the internal load distribution and various other stress and dynamic parameters.

The last section of data provides the complete stiffness matrix.

OUTPUT DATA FOR RUN NO. 1 BEARING NO. 1

BALL NUMBER	BALL AZIMUTH DEG	CONTACT LOAD OUTER LB	CONTACT LOAD INNER LB	CONTACT ANGLE OUTER DEG	CONTACT ANGLE INNER DEG	CONTACT AREA LENGTH OUTER IN	CONTACT AREA LENGTH INNER IN	CONTACT AREA WIDTH OUTER IN	CONTACT AREA WIDTH INNER IN	CENTRIFUGAL FORCE LB
4	1.2000E+02	1.7117E+02	1.7117E+02	0.0000	0.0000	6.7266-02	8.9588-02	1.1749-02	8.3812-03	2.0415-03
5	1.6000E+02	4.4102E+02	4.4101E+02	0.0000	0.0000	9.2217-02	1.2213-01	1.6105-02	1.1491-02	2.0444-03
6	2.0000E+02	4.4102E+02	4.4101E+02	0.0000	0.0000	9.2217-02	1.2213-01	1.6105-02	1.1491-02	2.0444-03
7	2.4000E+02	1.7117E+02	1.7117E+02	0.0000	0.0000	6.7266-02	8.9588-02	1.1749-02	8.3812-03	2.0415-03

BALL NUMBER	MEAN COMPRESSIVE STRESS OUTER PSI	MEAN COMPRESSIVE STRESS INNER PSI	CONTACT DEFLECTION OUTER IN	CONTACT DEFLECTION INNER IN	H/D VALUE OUTER	H/D VALUE INNER	TYPE OF CONTROL	ROTATIONAL VELOCITY RPM	ORBITAL VELOCITY RPM	GYROSCOPIC MOMENT LB*IN
4	0.0000	2.9188+05	4.9454-04	4.4556-04	3.1277-02	5.7942-02	INNER	-2.3686+03	3.9878+02	0.0000
5	0.0000	4.0010+05	9.2945-04	8.3737-04	6.0503-02	1.1576-01	INNER	-2.3710+03	3.9887+02	0.0000
6	0.0000	4.0010+05	9.2945-04	8.3737-04	6.0503-02	1.1576-01	INNER	-2.3710+03	3.9887+02	0.0000
7	0.0000	2.9188+05	4.9454-04	4.4556-04	3.1277-02	5.7942-02	INNER	-2.3686+03	3.9878+02	0.0000

REACTIONS OF BEARING ON SHAFT ALONG X LB	ALONG Y LB	ALONG Z LB	ABOUT X LB*IN	ABOUT Y LB*IN	TOTAL DISPLACEMENT OF INNER WITH RESPECT TO OUTER ALONG X IN	ALONG Y IN	ALONG Z IN	ABOUT X RADIANS	ABOUT Y RADIANS
-1.3000+03	5.3406-05	0.0000	0.0000	0.0000	-1.8802-03	0.0000	0.0000	0.0000	0.0000

PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISPLACEMENTS

DFX/DX LB/IN	DFX/DY LB/IN	DFX/DZ LB/IN	DFX/DALX LB/RAD	DFX/DALY LB/RAD
7.9779+05	-4.8828-03	0.0000	0.0000	0.0000

DFY/DX LB/IN	DFY/DY LB/IN	DFY/DZ LB/IN	DFY/DALX LB/RAD	DFY/DALY LB/RAD
-4.8828-03	4.9727+05	0.0000	0.0000	0.0000

DFZ/DX LB/IN	DFZ/DY LB/IN	DFZ/DZ LB/IN	DFZ/DALX LB/RAD	DFZ/DALY LB/RAD
0.0000	0.0000	1.2067+05	-1.8311-04	-4.5468+04

DMX/DX LB*IN/IN	DMX/DY LB*IN/IN	DMX/DZ LB*IN/IN	DMX/DALX LB*IN/RAD	DMX/DALY LB*IN/RAD
0.0000	0.0000	-6.1035-05	7.9715+03	3.2043-04

DMY/DX LB*IN/IN	DMY/DY LB*IN/IN	DMY/DZ LB*IN/IN	DMY/DALX LB*IN/RAD	DMY/DALY LB*IN/RAD
0.0000	0.0000	-4.5468+04	4.4250-04	1.8187+04

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Figure 15 Output Data for Sample Problem

SECTION IV
DESIGN DATA

Three separate sets of design charts were included in the original Part IV report. These were:

a. Pure Radial Loaded Bearings (Deep Groove) Contact Angle $\beta' = 0^{\circ}$

b. Pure Thrust Loaded Bearings (Deep Groove) Contact Angle $\beta' = 0^{\circ}$

c. Angular Contact Bearings with Axial Preload and Applied Radial Load $\beta' = 25^{\circ}, 15^{\circ}$

As a check of these design charts shows some differences between the original program prediction and the present program prediction, the design charts have been recalculated and are shown in Appendices B, C, and D.

Table 2 describes the dimensions and symbols used for the deep-grooved ball bearings. Table 3 contains information pertaining to the angular contact bearings. These bearings are identical to those studied previously in Reference 1.

4.1 Radial Stiffness Versus Radial Load

The first set of three charts (Appendix B) contains graphs of radial stiffness versus radial load. Load levels are indicated on the curves. The effects of bearing size and race curvatures are illustrated by these four charts. In general, a bearing with curvatures of $f_1 = .530$, $f_2 = .516$ is stiffer than the same bearing operating with curvatures of $f_1 = f_2 = .570$, for the same radial load. Radial stiffness is higher for a bearing with a larger bore diameter and/or a greater number of balls. Note, for pure radial load, the linear relationship between log K_{yy} and log F_y. This was previously illustrated in Figure 6.

TABLE 2

DEEP GROOVE BEARINGS

Bearing Symbol	Bore (Inch)	Bore mm	O.D. (Inch)	Ball Diameter, Inch	Number of Balls	f_i	f_o
A1	.5906	15	1.2598	.1875	9	.516	.530
A2	.5906	15	1.2598	.1875	9	.570	.570
B1	.9843	25	1.8504	.250	10	.516	.530
B2	.9843	25	1.8504	.250	10	.570	.570
C1	1.378	35	2.4409	.3125	11	.516	.530
C2	1.378	35	2.4409	.3125	11	.570	.570
D1	2.1654	55	3.5433	.40625	13	.516	.530
D2	2.1654	55	3.5433	.40625	13	.570	.570
E1	2.9528	75	4.5276	.46875	15	.516	.530
E2	2.9528	75	4.5276	.46875	15	.570	.570
AA1	.5906	15	1.378	.2345	8	.516	.530
AA2	.5906	15	1.378	.2345	8	.570	.570
BB1	.9843	25	2.0472	.3125	9	.516	.530
BB2	.9843	25	2.0472	.3125	9	.570	.570
CC1	1.378	35	2.8346	.4375	9	.516	.530
CC2	1.378	35	2.8346	.4375	9	.570	.570
DD1	2.1654	55	3.937	.5625	10	.516	.530
DD2	2.1654	55	3.937	.5625	10	.570	.570
EE1	2.9528	75	5.1181	.6875	11	.516	.530
EE2	2.9528	75	5.1181	.6875	11	.570	.570

TABLE 3

ANGULAR CONTACT BEARINGS

$\beta_o = 15^o, 25^o$

Basic Static Load (lb)	Bearing Number	Bore (Inch)	Bore mm	O.D. (Inch)	d (in)	Number of Balls	f_i	f_o	Axial Preload (lb)		
									S.L.	M.	P.H.
630	PA 1 PA 2	.5906	15	1.2598	.1875	11	.516 .570	.530 .570	20	50	100
1400	PB 1 PB 2	.9843	25	1.8504	.2500	13	.516 .570	.530 .570	50	100	200
2600	PC 1 PC 2	1.3780	35	2.4409	.3125	15	.516 .570	.530 .570	50	100	200
5100	PD 1 PD 2	2.1654	55	3.5433	.40625	18	.516 .570	.530 .570	100	200	300
8600	PE 1 PE 2	2.9528	75	4.5276	.46925	21	.516 .570	.530 .570	100	200	300
760	PAA1 PAA2	.5906	15	1.3780	.23425	10	.516 .570	.530 .570	20	50	100
1640	PBB1 PBB2	.9843	25	2.0472	.3125	12	.516 .570	.530 .570	50	100	200
3750	PCC1 PCC2	1.378	35	2.8346	.4375	12	.516 .570	.530 .570	50	100	200
7300	PDD1 PDD2	2.1654	55	3.9370	.5625	14	.516 .570	.530 .570	100	200	300
12200	PEE1 PEE2	2.9528	75	5.1181	.6875	16	.516 .570	.530 .570	100	200	300

4.2 Axial Stiffness Versus Thrust Load

The second set of eight charts (Appendix C) contains graphs of axial stiffness and axial deflection versus axial thrust applied load. Load levels are tabulated in Table 4 for bearings undergoing a pure thrust load.

A similar observation as given above for radially loaded bearings, can be made for the thrust loaded bearing, i.e., a bearing operating with curvatures of $f_1 = .530$, $f_2 = .516$, is stiffer than the same bearing operating with curvatures of $f_1 = f_2 = .570$ for the same axial load. For all practical purposes, however, an average curve may be drawn for axial stiffness versus axial load for all bearing sizes. In practicular, the bearing with the smaller bore and less balls is less stiff at light loads and more stiff at heavy loads as compared to the larger bore bearing. There is an approximate linear relationship between log K_{zz} and log F_z (also see Figure 8).

4.3 Radial Stiffness Versus Radial Load with Preload

The third set of (24) charts (Appendix D) contain graphs of radial stiffness versus radial load with preload. Load levels are indicated on the curves. The effects of bearing size, race curvatures, initial contact angle, and axial preload are illustrated by these 24 charts. For the same radial load and axial preload, a bearing operating with curvatures of $f_1 = .530$, $f_2 = .516$, is stiffer than the same bearing operating with curvatures of $f_1 = f_2 = .570$. The radial stiffness level is higher for a bearing with a larger bore diameter and/or a greater number of balls, and the smaller initial contact angle, ($\beta' = 15^\circ$). In general, the radial stiffness vs. radial load curve for an angular contact bearing is composed of three different behaving regions. One region shows the stiffness to be constant with varying radial load. (This is the light radial load region.) The middle, or moderate radial load region shows a minimum value for radial stiffness. The heavily radial loaded region shows a linear relationship between log K_{xx} and log F_x. This third region is similar in behavior to that of the characteristics of a pure radial loaded deep grooved bearing. The basic cause for this curve having three separate regions is due to the axial preload. In region one, the axial preload has a great effect in holding the radial stiffness constant. In region two, where the applied radial

TABLE 4

AXIAL LOADED DEEP GROOVE BEARINGS

Table of approximate load values corresponding to C/P = 5 and C/P = 10 load levels.

Bearing Symbol	Load (lb)	
	C/P = 5	C/P = 10
A1	290	100
B1	600	250
C1	1100	450
D1	2250	950
E1	3650	1500
A2	70	30
B2	175	75
C2	300	100
D2	550	250
E2	950	350
AA1	380	155
BB1	800	300
CC1	1550	650
DD1	3000	1250
EE1	5050	2100
AA2	95	50
BB2	200	100
CC2	400	200
DD2	1500	350
EE2	2550	700

load becomes equal in magnitude to the axial preload the radial stiffness tends to decrease with increasing applied radial load to a minimum value. In the third region, the axial preload has little or no effect, and the angular contact bearing reflects the behavior of a pure radially loaded bearing i.e., a linear log K_{xx} versus log F_x relationship.

Thus another point one is led to observe is the role of axial preload magnitude on the three regions of a typical stiffness versus load curve. Three different preloads are represented in these charts and are tabulated in Table 3. These preloads are given the names selected light, moderate, and preferred heavy. The effect of increased preload is to increase the region one load range and decrease region three load range. Thus, the ultimate is a constant radial stiffness with varying radial load obtained with an infinite preload. The increased preload also has the effect of increasing the level of stiffness in regions one and two. However, it should be noted particularly that the level of stiffness in region three, for the same radial load, is the same for all preload values. This, as mentioned above, is because the axial preload effect is relieved entirely above a certain (radial load) (axial preload) ratio. (Approximately $F_x/F_z = 3$ for $\beta' = 25^\circ$ and $F_x/F_z = 4$ for $\beta' = 15^\circ$.)

In general, the light and extra light deep grooved ball bearings examined here will have a radial stiffness ranging from 10^5 to 2×10^6 for radial loads of from 10 to 2,000 lbs. The angular contact bearings will have radial stiffness values from 2×10^5 to 2×10^6 for radial loads of from 10 to 2,000 lbs. The deep grooved ball bearings will have an axial stiffness per bearing of from 2×10^4 to 4×10^6 for thrust loads of from 10 to 10^4 lbs. As in the case of the preloaded radial bearing, preloading will increase these values of axial stiffness.

APPENDIX A

COMPUTER PROGRAM FOR CALCULATING THE STIFFNESS MATRIX OF A BALL BEARING

```
C      MAIN PROGRAM
C  CARD  COLS.   ITEM
C    1     1-10   NUMBER OF BALLS
C         11-20   BALL DIAMETER - IN
C         21-30   PITCH DIAMETER - IN
C         31-40   CONTACT ANGLE - DEGREES
C         41-50   OUTER-RACE CURVATURE FACTOR (AS .52)
C         51-60   INNER-RACE CURVATURE FACTOR (AS .52)
C         61-70   IF CONTACT ANGLE IS ZERO  THIS ITEM IS THE TOTAL MOUNTED
C                 DIAMETRAL CLEARANCE FROM WHICH THE MOUNTED CONTACT
C                 ANGLE WILL BE CALCULATED. IF CONTACT ANGLE IS POSITIVE
C                 IT IS THE INCREMENT IN INTERNAL CLEARANCE FROM WHICH THE
C                 MOUNTED CONTACT ANGLE WILL BE CALCULATED. AN INCREASE IN
C                 CLEARANCE IS POSITIVE. A REDUCTION IN CLEARANCE IS
C                 NEGATIVE.
C    2     1      PUNCH 1 (NO DECIMAL POINT)
C          2-80   TITLE CARD - PUNCH ANYTHING.
C    3     1      LEAVE BLANK.
C          2-80   SECOND TITLE CARD - PUNCH ANYTHING.
C    4     1-10   POISSON'S RATIO FOR OUTER RING. IF BLANK PROGRAM ASSUMES
C                 .25
C         11-20   POISSON'S RATIO FOR INNER RING. IF BLANK PROGRAM ASSUMES
C                 .25
C         21-30   POISSON'S RATIO FOR BALLS. IF BLANK PROGRAM ASSUMES .25
C         31-40   MODULUS OF ELASTICITY FOR OUTER RING. IF BLANK PROGRAM
C                 ASSUMES 29.E6
C         41-50   MODULUS OF ELASTICITY FOR INNER RING. IF BLANK PROGRAM
C                 ASSUMES 29.E6
C         51-60   MODULUS OF ELASTICITY FOR BALLS. IF BLANK PROGRAM ASSUMES
C                 29.E6
C         61-70   BALL DENSITY - LB/IN**3. IF BLANK PROGRAM ASSUMES .283
C         71-80   IF INNER-RACE CONTROL IS SUSPECTED PUNCH 2. HERE. IF
C                 BLANK PROGRAM ASSUMES 1. CORRESPONDING TO OUTER-RACE
C                 CONTROL. IF ASSUMPTION IS INCORRECT PROGRAM WILL CORRECT
C                 IT.
C    5     1-10   RPM OF OUTER RING
C         11-20   RPM OF INNER RING
C         21-30   LOAD APPLIED TO INNER RING ALONG X - LB
C         31-40   LOAD APPLIED TO INNER RING ALONG Z - LB. MUST BE
C                 NEGATIVE
C         41-50   INITIAL DISPLACEMENT OF INNER RING ALONG X - IN
C         51-60   INITIAL DISPLACEMENT OF INNER RING ALONG Y - IN
C         61-70   INITIAL DISPLACEMENT OF INNER RING ALONG Z - IN.
C                 A POSITIVE VALUE LOADS THE BEARING IN THRUST
C         71-80   INITIAL DISPLACEMENT OF INNER ABOUT X - RADIANS
C    6     1-10   INITIAL DISPLACEMENT OF INNER ABOUT Y - RADIANS
C         11-20   A 1. HERE PERMITS DEFLECTION ALONG X
C         21-30   A 1. HERE PERMITS DEFLECTION ALONG Y
C         31-40   A 1. HERE PERMITS DEFLECTION ALONG Z
C    NOTE         IF INITIAL DISPLACEMENTS EXIST ALONG X, Y OR Z
C                 DEFLECTIONS IN THOSE MODES DUE TO EXTERNAL LOADS ARE
```


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```
C      PREVENTED
C
C      IF EXTERNAL LOADS ALONG X AND/OR  Z EXIST DEFLECTIONS IN
C      THOSE MODES ARE PERMITTED
C
C      DO NOT ENTER BOTH INITIAL DISPLACEMENT AND EXTERNAL LOAD
C      FOR THE SAME MODE
C
C      TO RUN ADDITIONAL LOADS WITH SAME BEARING REPEAT CARDS 5
C      AND 6 DIRECTLY AFTER LAST CARD 6
C
C      TO RUN AN ADDITIONAL BEARING PLACE TWO BLANKS AFTER LAST
C       CARD 6 AND READ CARDS 1 ET SEQ.
C
C      PLACE THREE BLANKS AFTER LAST CARD 6 TO STOP
      COMMON  A(3,3),AMAJ(2,60),B(2),BETA,BD,BET,BEIM,BBET,BT(2,60),BMI
     1N(2,60),BREV(60),C(2),CRV(2),COSB,COR(3),CB(2),CSEP(2),CBAL(2),D,
     2DENS,DFL11(5),DFL(5),CTV(5,5),DPX(2,2),DEL(2),DDFLX(2,2),DAAUG(2),
     3DCX(2,2),DPX(2,2),DALY(2),DCX(2),DGYRO(2),DPSIX(2,2),DDELA(2,2),
     4DPA(2,2),DGA(2,2),DGY(2),DPA(2,2),DPSIA(2),DXA(2),E,EL(2),ER(3),
     5FREV(60),FSAVE(2),F(2),FREE(3),G2(2),GG(2),H(2),HH(2),H1(2),H2(2),
     6HOD(2),IBP,ILOAD,IQUIT,ICTL(60),IC,JPASS,N,NOLOAD,NZ,PU,PR(2),PD,
     7PDM,PHLF(60),P(2,60),PX(2),PSI(2),R,RPY(2),ROT(2),SIN3,STRS(2),SB(
     82),SNEF(2),SO(2),SBAL(2),        TSAVE(2),TRQ(2),V1(2),V2(2),WQTH(2),
     9XN,XMASS,YIN,XF(3),XF1(5),XCX(60),ADEL(2,60),AGYRO(60),X(2),XK(2)
      COMMON DPD(2,5),DEX(2),YR(2), YAL(60),TOL(5)
      DIMENSION AA(3,3),CORR(3),ERR(3),WORD(2)
      DATA WORD(1),WORD(2)/4H OUT,4H INN/
      TOL(1)=1.E-7
      TOL(2)=TOL(1)
      TOL(3)=1.E-3
      TOL(4)=TOL(3)
      TOL(5)=TOL(3)
      C(1)=1.
      C(2)=-1.
      DAD(1,1)=1.
      DAD(1,2)=0.
      DAD(1,3)=0.
      DAD(2,1)=0.
      H(2)=0.
      HH(1)=0.
      H1(1)=1.
       H1(2)=0.
       H2(1)=0.
      H2(2)=1.
      DAD(2,4)=0.
      DAD(2,5)=0.
       DEX(1)=0.
      DEX(2)=2.
       DBA(1,1)=0.
      DBA(1,2)=0.
```

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      DGA(1,1)=0.
      DGA(1,2)=0.
      DDELA(1,1)=0.
      DDELA(1,2)=0.
      IBR=0
10    READ(5,20)XN,D,E,BETA,F(1),F(2),PD
20    FORMAT(8E10.0)
      IF(XN.EQ.0.)STOP
      N=XN
      ISTOP=0
      READ(5,30)
30    FORMAT(1H1,79H
     1                              /1X,79H
     2                                    )
      WRITE(6,30)
      READ(5,20)PR(1),PR(2),PB,YR(1),YR(2),YB,DFNS,XIC
      IF(XIC.EQ.0.)XIC=1.
      DO 35 J=1,N
35    ICTL(J)=XIC
      ILOAD=0
      IBR=IBR+1
      BD=(F(1)+F(2)-1.)*D
      DO 40 K=1,2
40    CRV(K)=(F(K)-.5)*D
      BET=BETA/57.295780
      TEMP=COS(BET)
      A1=2.*LD*(1.-TEMP)
      PDX=0.
      PDM=A1+PD
      IF(PDM-2.*BD)70,50,50
50    WRITE(6,60)
60    FORMAT(47H0  THIS BEARING HAS TOO MUCH INTERNAL LOOSENESS)
      ISTOP=1
      GO TO 275
70    IF(PDM)75,75,80
75    BETM=0.
      SINB=0.
      COSB=1.
      PDX=PDM
      GO TO 90
80    BETM=ACOS((2.*BD-PDM)/(2.*BD))
      SINB=SIN(BETM)
      COSB=COS(BETM)
90    BETM=BETM*57.29578
100   DO 140 K=1,2
      IF(YR(K))110,110,120
110   YR(K)=29.E6
120   IF(PR(K))130,130,140
130   PR(K)=.25
140   CONTINUE
      B=.5*E+CRV(2)*COSB
      IF(YB)150,150,160
```


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```
      YB=29.E6
      IF(PB)170,170,180
  170 PB=.25
  180 DO 190 K=1,2
  190  EL(K)=4.*((1.-PR(K)**2)/YR(K)+(1.-PB**2)/YB)
      IF(DENS)220,220,230
  220 DENS=.283
  230 XMASS=1.3550693E-3*DENS*D**3
      XIN=XMASS*.1*D**2
      WRITE(6,240)IBR
  240 FORMAT(30H0  DESIGN DATA FOR BEARING NO.,I3)
      WRITE(6,250)
  250 FORMAT(119H0   NO. OF        BALL        PITCH      INITIAL CONTACT A
     1NGLE        RACE CURVATURE          INTERNAL CLEARANCE     MATERIAL/1
     118H    BALLS      DIAMETER      DIAMETER      DESIGN      MOUNTED
     2 OUTER        INNER      INCREMENT    OPERATING     DENSITY/18X,2HIN,1
     30X,2HIN,9X,3HDEG,9X,3HDEG,34X,2HIN,10X,2HIN,7X,8HLB/IN**3)
      WRITE(6,260)XN,D,E,BETA,BETM,F(1),F(2),PD,PDM,DENS
  260 FORMAT(1P11E12.4)
      WRITE(6,270)
  270 FORMAT(62H0        MODULUS OF ELASTICITY                        POISSON'S
     1 RATIO/69H    OUTER        INNER        BALLS        OUTER        INNE
     2R        BALLS/35H    LB/IN**2     LB/IN**2     LB/IN**2)
      WRITE(6,260)YR(1),YR(2),YB,PR(1),PR(2),PB
  275 READ (5,20)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(1),
     1DFL11(4),DFL11(5),FREE(2),FREE(3),FREE(1)
      IF(ABS(RPM(1))+ABS(RPM(2)).EQ.0.)GO TO 10
      IF(ISTOP.EQ.1)GO TO 275
      IOUT=0
      ILOAD=ILOAD+1
      WRITE(6,280)ILOAD,IBR
  280 FORMAT(25H1   INPUT DATA FOR RUN NO.,I3,12H BEARING NO.,I3)
      WRITE(6,290)
  290 FORMAT(105H0   RPM OF       RPM OF     LOADS APPLIED TO SHAFT      IN
     1ITIAL DISPLACEMENTS OF INNER WITH RESPECT TO OUTER/106H    OUTER
     2     INNER        ALONG X      ALONG Z      ALONG X      ALONG Y     AL
     3ONG Z     ABOUT X      ABOUT Y/30X,76HLB                 LB           IN
     4          IN           IN           RADIANS      RADIANS)
      WRITE(6,260)RPM(1),RPM(2),XF(2),XF(1),DFL11(2),DFL11(3),DFL11(1),
     1DFL11(4),DFL11(5)
      WRITE(6,300)
  300 FORMAT(31H0      1. = DEFLECTION PERMITTED/34H    ALONG X      ALONG
     1Y      ALONG Z)
      IF(XF(1).NE.0.)FREE(1)=1.
      IF(XF(2).NE.0.)FREE(2)=1.
      DO 305 K=1,3
      IF(DFL11(K).NE.0.)FREE(K)=0.
  305 CONTINUE
      WRITE(6,260)FREE(2),FREE(3),FREE(1)
      RPM(1)=RPM(1)*.1047198
      RPM(2)=RPM(2)*.1047198
      KKK=0
```


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```
      DO 310 K=1,3
      IF(ERR(K).GT.0.)KKK=KKK+K**2
  310 CONTINUE
      DEL(1)=0.
      DEL(2)=0.
      DEL(3)=0.
      IF(XF(1).LT.0.)DEL(1)=.005*D/(1.+SINB)
      TEMP=.005*D/COSB
      IF(XF(2).GE.0.)DEL(2)=-SIGN(TEMP,XF(2))
      DO 330 ITER=1,20
      NOLOAD=0
      DO 320 K=1,5
      XF1(K)=0.
      DO 320 L=1,5
  320 DTV(L,K)=0.
      DO 360 J=1,N
      XJ=J
      PHI=6.283185*(XJ-1.)/XN
      PHEL(J)=PHI
      SPH=SIN(PHI)
      CPH=COS(PHI)
      JPASS=0
      CALL BALL(SPH,CPH,PDX)
      IF(IQUIT)350,360,330
  330 KK=340
  340 WRITE(6,350)KK,ITER,J,(DEL(K),K=1,3)
  350 FORMAT(13H0MAIN PROGRAM,3I4,1P3E12.4)
      GO TO 375
  360 CONTINUE
      IF(NKK-1)550,370,380
  370 COR1=(XF1(1)+XF(1))/DTV(1,1)
      DEL(1)=DEL(1)-COR1
      IF(ABS(COR1)-TOL(3))550,530,530
  380 IF(NKK.GT.4)GO TO 390
      ER2=XF1(2)+XF(2)
      COR2=ER2/DTV(2,2)
      DEL(2)=DEL(2)-COR2
      IF(ABS(COR2)-TOL(4))550,530,530
  390 IF(NKK.GT.5)GO TO 410
      ER1=XF1(1)+XF(1)
      ER2=XF1(2)+XF(2)
      DET=DTV(1,1)*DTV(2,2)-DTV(2,1)*DTV(1,2)
      COR1=(ER1*DTV(2,2)-ER2*DTV(1,2))/DET
      COR2=(DTV(1,1)*ER2-DTV(2,1)*ER1)/DET
      DEL(1)=DEL(1)-COR1
      DEL(2)=DEL(2)-COR2
      IF(ABS(COR1)-TOL(3))400,530,530
  400 IF(ABS(COR2)-TOL(4))550,530,530
  410 IF(NKK.GT.9)GO TO 420
      COR3=XF1(3)/DTV(3,3)
      DEL(3)=DEL(3)-COR3
      IF(ABS(COR3)-TOL(5))550,530,530
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```
420  IF(KKK.GT.10)GO TO 440
     ER1=XF1(1)+XF(1)
     ER3=XF1(3)
     DET=DTV(1,1)*DTV(3,3)-DTV(3,1)*DTV(1,3)
     CORR1=(ER1*DTV(3,3)-ER3*DTV(1,3))/DET
     CORR3=(DTV(1,1)*ER3-DTV(3,1)*ER1)/DET
     DFL(1)=DFL(1)-COR1
     DFL(3)=DFL(3)-COR3
     IF(ABS(COR1)-TOL(3))430,530,530
430  IF(ABS(COR3)-TOL(5))550,530,530
440  IF(KKK.GT.13)GO TO 460
     ER2=XF1(2)+XF(2)
     ER3=XF1(3)
     DET=DTV(2,2)*DTV(3,3)-DTV(3,2)*DTV(2,3)
     CORR2=(ER2*DTV(3,3)-ER3*DTV(2,3))/DET
     CORR3=(DTV(2,2)*ER3-DTV(3,2)*ER2)/DET
     DFL(2)=DFL(2)-COR2
     DFL(3)=DFL(3)-COR3
     IF(ABS(COR2)-TOL(4))450,530,530
450  IF(ABS(COR3)-TOL(5))550,530,530
460  KO=
     ERR(1)=XF1(1)+XF(1)
     ERR(2)=XF1(2)+XF(2)
     ERR(3)=XF1(3)
     DO 470 L=1,3
     DO 470 M=1,3
470  AA(M,L)=DTV(M,L)
     CALL SIMUL1(AA,KO,ERR,CORR,IQUIT)
     IF(IQUIT)500,500,480
480  WRITE(6,490)J,ITER,(DFL(K),K=1,3)
490  FORMAT(17H MAIN PROGRAM 490,2I6,1P3E12.4)
     GO TO 275
500  DO 510 K=1,3
510  DFL(K)=DFL(K)-CORR(K)
     DO 520 K=1,3
     IF(ABS(CORR(K))-TOL(K+2))520,530,530
520  CONTINUE
     GO TO 550
530  CONTINUE
     WRITE(6,540)
540  FORMAT(17H MAIN PROGRAM 540)
     GO TO 275
550  WRITE(6,560)ILOAD,IBB
560  FORMAT(26H   OUTPUT DATA FOR RUN NO.,I3,12H BEARING NO.,I3)
     WRITE(6,570)
570  FORMAT(130H       BALL              BALL              CONTACT LOAD          C
    1ONTACT ANGLE            CONTACT AREA LENGTH       CONTACT AREA WIDTH
    2CONTACT ANGLE/129H     NUMBER          AZIMUTH          OUTER          INNER
    3  OUTER           INNER           OUTER           INNER          OUTER          INNE
    60           FORCE/17X,3HDEG,10X,2HLB,10X,2HLB,9X,3HDEG,9X,3HDEG,10X,2H
    7IN,10X,2HIN,10X,2HIN,10X,2HIN,10X,2HLB)
     DO 590 J=1,
```

```
      IF(P(2,J).LE.0.)GO TO 590
      PHI=PHIE(J)*57.29578
      WRITE(6,580)J,PHI,P(1,J),P(2,J),BT(1,J),BT(2,J),AMAJ(1,J),AMAJ(2,J
     1),BMIN(1,J),BMIN(2,J),XCX(J)
  580 FORMAT(I7,5X,1P10E12.4)
  590 CONTINUE
      WRITE(6,600)
  600 FORMAT(132H0     BALL     MEAN COMPRESSIVE STRESS     CONTACT DEFLECT
     1ION          H/D VALUE          TYPE OF     ROTATIONAL    ORBITAL
     2 GYROSCOPIC/130H    NUMBER          OUTER          INNER          OUTER
     3  INNER          OUTER          INNER          CONTROL      VELOCITY     VELOC
     4ITY      MOMENT/17X,3HPSI,9X,3HPSI,10X,2HIN,10X,2HIN,45X,3HRPM,9X,3
     5HRPM,8X,5HLB*IN)
      DO 630 J=1,N
      IF(P(2,J).LE.0.)GO TO 630
      WRD=WORD(1)
      IF(ICTL(J).EQ.2)WRD=WORD(2)
      DO 610 K=1,2
      STRS(K)=1.273240*P(K,J)/(AMAJ(K,J)*BMIN(K,J))
      TEMP=ASIN(.5*AMAJ(K,J)/(F(K)*D))+BT(K,J)/57.29578
  610 HOD(K)=F(K)*(1.-COS(TEMP))
      WRITE(6,620)J,STRS(K),STRS(2),XDEL(1,J),XDEL(2,J),HOD(1),HOD(2),
     1WRD,BREV(J),EREV(J),XSYRD(J)
  620 FORMAT(I7,5X,1P6E12.4,3X,A4,2HER,3X,1P2E12.4)
  630 CONTINUE
      WRITE(6,640)
  640 FORMAT(1H0,15X,29HREACTIONS OF BEARING ON SHAFT,21X,49HTOTAL DISPL
     1ACEMENT OF INNER WITH RESPECT TO OUTER/119H    ALONG X        ALONG Y
     2     ALONG Z        ABOUT X        ABOUT Y        ALONG X        ALONG Y        AL
     3ONG Z        ABOUT X        ABOUT Y/118H       LB             LB             LB
     4          LB*IN          LB*IN            IN             IN             IN
     5 RADIANS      RADIANS)
      DO 650 K=1,3
  650 DEL(K)=DEL(K)+DEL11(K)
      WRITE(6,260)XF1(2),XF1(3),XF1(1),XF1(4),XF1(5),DEL(2),DEL(3),DEL(1
     1),DEL11(4),DEL11(5)
      WRITE(6,660)
  660 FORMAT(65H0  PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISP
     1LACEMENTS)
      WRITE(6,670)
  670 FORMAT(59H0    DFX/DX         DFX/DY         DFX/DZ        DFX/DALX      DFX/D
     1ALY)
      WRITE(6,680)
  680 FORMAT(58H     LB/IN          LB/IN          LB/IN          LB/RAD        LB/R
     1AD)
      WRITE(6,260)DTV(2,2),DTV(2,3),DTV(2,1),DTV(2,4),DTV(2,5)
      WRITE(6,690)
  690 FORMAT(59H0    DFY/DX         DFY/DY         DFY/DZ        DFY/DALX      DFY/D
     1ALY)
      WRITE(6,630)
      WRITE(6,260)DTV(3,2),DTV(3,3),DTV(3,1),DTV(3,4),DTV(3,5)
      WRITE(6,700)
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```
!     700  FORMAT(59H0    DFZ/DY       DFZ/DY       DFZ/DZ       DFZ/DALX     DFZ/D
     1ALY)
           WRITE(6,630)
           WRITE(6,260)DTV(1,2),DTV(1,3),DTV(1,1),DTV(1,4),DTV(1,5)
           WRITE(6,710)
     710   FORMAT(59H0    DMX/DX       DMX/DY       DMX/DZ       DMX/DALX     DMX/D
     1ALY)
           WRITE(6,720)
     720   FORMAT(59H    LB*IN/IN     LB*IN/IN     LB*IN/IN     LB*IN/RAD    LB*IN/
     1RAD)
           WRITE(6,260)DTV(4,2),DTV(4,3),DTV(4,1),DTV(4,4),DTV(4,5)
           WRITE(6,730)
     730   FORMAT(59H0    DMY/DX       DMY/DY       DMY/DZ       DMY/DALX     DMY/D
     1ALY)
           WRITE(6,720)
           WRITE(6,260)DTV(5,2),DTV(5,3),DTV(5,1),DTV(5,4),DTV(5,5)
           GO TO 275
           END
           SUBROUTINE BALL(SPH,CPH,PDX)
            COMMON A(3,3),AMAJ(2,60),B(2),BETA,BD,BET,BETM,BBET,BT(2,60),BMI
     1N(2,60),BREV(60),C(2),CRV(2),COSB,COR(3),CB(2),CSEP(2),CBAL(2),D,
     2DENS,DFL11(5),DFL(3),DTV(5,5),DPX(2,2),DEL(2),DDFLX(2,2),DXNOG(2),
     3DGX(2,2),DPA(2,2),DALX(2),DCX(2),DGYRO(2),DPSIX(2,2),DDELA(2,2),
     4DBA(2,2),DGA(2,2),DGYA(2),DPA(2,2),DPSIA(2),DXA(2),E,EL(2),ER(3),
     5EREV(60),ESAVE(2),F(2),FREE(3),G2(2),GC(2),H(2),PH(2),H1(2),H2(2),
     6HOD(2),IBP,ILOAD,IQUIT,ICTL(60),IC,JPASS,N,NOLOAD,NZ,PD,PR(2),PS,
     7PDM,PHEE(60),P(2,60),PX(2),PSI(2),R,RPM(2),ROT(2),SIDE,STRS(2),SB(
     B2),SNEP(2),SQ(2),SBAL(2),    TSAVE(2),TRQ(2),V1(2),V2(2),WOTH(2),
     9XN,AMASS,XIN,XF(3),XF1(5),XCX(60),XDEL(2,60),XGYRO(60),X(2),XK(2)
            COMMON DAD(2,5),DEX(2),YP(2),  XAL(60),TOL(5)
           J=JPASS
           DAD(1,4)=B*SPH
           DAD(1,5)=B*CPH
           DAD(2,4)=CPH
           DAD(2,5)=SPH
           ICNT=0
           LOOP=0
           IC=ICTL(J)
           A1=BD*SINP+DFL(1)+DFL11(1)+R*(DFL11(4)*SPH+DFL11(5)*CPH)                BALL  22
           A2=BD*COSP+(DFL(2)+DFL11(2))*CPH+(DFL(3)+DFL11(3))*SPH-PDX*.5
           A1A2=A1**2+A2**2
           SQT=SQRT(A1A2)
     5     TEMP=A1
           CVRG2=0.
           IF(ABS(A1)-1.E-6)30,30,10
     10    IF(A1)20,20,50
     20    TG=-1.
           GO TO 60
     30    IF(A2-BD-1.E-8)70,40,40
     40    X(1)=0.
           X(2)=.55*(A2-BD)+CRV(1)
           GO TO 190
```


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```
50    TG=1.
60    IF((A2-CRV(1))**2+A1**2-CRV(2)**2)70,70,80
70    NOLOAD=NOLOAD+1
      P(2,J)=0.
      RETURN
80    TB=ABS(A1)/A2
      A1=ABS(A1)
      IF(SQT-BD)130,90,150
90    AX=A1*CRV(1)/SQT
      AY=A2*CRV(1)/SQT
100   XX=(AX*TB+AY-A2)/TB
      IF(XX)140,110,110
110   X(1)=(XX+.6*(AX-XX))*TG
      X(2)=A2-.6*(A2-AY)
120   A1=TEMP
      GO TO 190
130   CLM=(CRV(1)**2+A1A-CRV(2)**2)/(2.*CRV(1)*SQT)
      ALAM=ATAN(SQRT(1.-CLM**2)/CLM)
      B1=ATAN(TB)-ALAM
      AX=CRV(1)*SIN(B1)
      AY=SQRT(CRV(1)**2-AX**2)
      GO TO 100
140   X(1)=.6*AY*TG
      X(2)=AY+.4*AX*TB
      GO TO 120
150   DO 180 L=1,2
      G=D*SQRT(1./(1.+TB**2))*C(L)/E
      T1=1./E(L)-2.*G/(1.+G)
      T2=4.-2./E(L)+T1
      CALL ELIPIN(T1,T2,EE,EK,CSE2,IQUIT)
      IF(IQUIT)160,180,160
160   WRITE(6,170)J,L
170   FORMAT(11HOBALL - 170,2I4)
      RETURN
180   XK(L)=11.847711/EL(L)*SQRT(EE*D/(EK**3*CSE2*T2))
      APIN=(SQT-BD)/(1.+(XK(2)/XK(1))**.66666667)
      APO=SQT-BD-APIN
      AY=(CRV(1)+APO)*TB/SQRT(1.+TB**2)
      AY=SQRT((CRV(1)+APO)**2-AY**2)
      GO TO 100
190   DO 360 IT=1,20
      CVRG2=CVRG2+.2
      IF(CVRG2-1.)210,210,200
200   CVRG2=1.
210   B(1)=ATAN(X(1)/X(2))
      B(2)=ATAN((A1-X(1))/(A2-X(2)))
      DO 220 K=1,2
      SB(K)=SIN(B(K))
220   CB(K)=COS(B(K))
      DBX(1,1)=CB(1)**2/X(2)
      DBX(1,2)=-X(1)*DBX(1,1)/X(2)
      DBX(2,1)=-CB(2)**2/(A2-X(2))
```


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```
      DX(2,2)=-DX(2,1)*(A1-X(1))/(A2-X(2))
      E=E+2.*(X(2)-CRV(1)*COSB)
      DEL(1)=SQRT(X(1)**2+X(2)**2)-CRV(1)
      DEL(2)=SQRT((A1-X(1))**2+(A2-X(2))**2)-CRV(2)
      DO 230 K=1,2
      IF(DEL(K)-1.E-8)235,235,230
  230 CONTINUE
      GO TO 240
  235 NCLOAD=NOLOAD+1
      P(2,J)=0.
      RETURN
  240 DO 290 K=1,2
      G2(K)=C(K)/E2
      GG(K)=1.+C(K)*G2(K)
      T1=1./E(K)-2.*C(K)*G2(K)/GG(K)
      T2=4.-2./E(K)+T1
      CALL FLIPIN(T1,T2,EE,EK,CSE2,IQUIT)
      IF(IQUIT)270,270,250
  250 WRITE(6,260)J,K
  260 FORMAT(11H0OVALL - 260,2I4)
      RETURN
  270 ESAVE(K)=EE
      TSAVE(K)=T2
      CSEP(K)=SQRT(CSE2)
      SNEP(K)=SQRT(1.-CSE2)
      XK(K)=11.84771 /EL(K)*SQRT(D*EE/(EK**3*CSE2*T2))
      SG(K)=AK(K)*SQRT(DEL(K))
      DELX(K,1)=C(K)*SG(K)
      DELY(K,2)=C(K)*CR(K)
  280 CTAU=T1/T2
      STAU=SQRT(1.-CTAU**2)
      DTDG=2.*(T2-T1)*C(K)/((T2*GG(K))**2*STAU)
      DEDEP=(EE-EK*CSE2)/(SNEP(K)*CSEP(K))
      DEDEP=(EE-EK)/SNEP(K)*CSEP(K)
      DEDTAU=(EE*SNEP(K))**2*STAU*.5/(CSEP(K)*(-2.*EE*(EK-EE)/SNEP(K)+(E
     1E*DKDEP-EK*DEDEP)*CSEP(K)))
      DXKDEP=5.0238555*SQRT(D/(T2*EE*EK**5))/(EL(K)*CSE2)*((EK*DEDEP-3.
     1*EE*DKDEP)*CSEP(K)+2.*EK*EE*SNEP(K))
      DXKDT2=-XK(K)*.5/T2
  290 DXKDG(K)=DXKDEP*DEDTAU*DTDG-DXKDT2*2.*C(K)/GG(K)**2
      DO 300 L=1,2
      DO 300 M=1,2
      DGA(L,M)=-D/E2**2*(E2*SB(L)*DBX(L,M)+CB(L)*DEX(M))
  300 DBX(L,M)=1.5*SQ(L)*DDELY(L,M)+DEL(L)*SQRT(DEL(L))*DXKDG(K)*DGX(L,M
     1)
      TAL=SB(IC)/(CB(IC)+C(IC)*D/E2)
      ALX=ATAN(TAL)
      SAL=SIN(ALX)
      CAL=COS(ALA)
      DO 310 K=1,2
      TEMP=B(K)-ALX
      SBAL(K)=SIN(TEMP)
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```
310   CBAL(K)=COS(TEMP)
      T1=GG(1)*CBAL(2)
      T2=GG(2)*CBAL(1)
      DN=1.+T2
      REVB=E2*(RPM(1)-RPM(2))*GG(1)*GG(2)/(D*DN)
      REVE=(1.*RPM(1)+T2*RPM(2))/DN
      CX=AMASS*E2*REVE**2*.5
      GYRO=XIN*REVE*REVB*SAL
      H(1)=2.*GYRO/D
      HH(2)=H(1)
      Y1=CB(IC)**2+C(IC)*G2(IC)
      Y2=CB(IC)**2-SB(IC)**2
      Y3=SB(IC)*CB(IC)
      Y4=(CAL/Y1)**2
      Y5=E2*(RPM(1)-RPM(2))/(D*DN**2)
      Y6=REVE*D
      Y7=2./G*(H1(IC)*CB(1)-H2(IC)*CB(2))
      Y8=2./G*(H1(IC)*SB(1)-H2(IC)*SB(2))
      Y9=.5*G*Y7
      Y10=.5*D*Y8
      DO 320 K=1,2
      DALX(K)=(Y1*Y2*DBX(IC,K)-Y3*(-2.*Y3*DBY(IC,K)+C(IC)*DGX(IC,K)))*Y4
      V1(K)=GG(1)*SBAL(2)*(DBX(2,K)-DALX(K))
      V2(K)=GG(2)*SBAL(1)*(DBX(1,K)-DALX(K))
      DX(K)=DEL(K)*SG(K)
      DOMEX=.5*(GG*(-GG(1)*DGY(2,K)+GG(2)*DGX(1,K))-GG(1)*GG(2)*(-V1(K)+
     1CBAL(2)*DGX(1,K)-V2(K)-CBAL(1)*DGX(2,K)))+REVB*DFX(K)/E2
      DOMEY=(DN*(RPM(1)*(-V1(K)+CBAL(2)*DGX(1,K))-RPM(2)*(V2(K)+CBAL(1)*
     1DGX(2,K)))-Y6*(-V1(K)+CBAL(2)*DGX(1,K)-V2(K)-CBAL(1)*DGX(2,K)))/
     2DN**2
      DGX(K)=XMASS*(E2*REVE*DOMEX+.5*REVE**2*DFY(K))
320   DGYR0(K)=XIN*((REVB*DOMEX+REVE*DOMBX)*SAL+REVB*REVE*CAL*DALX(K))
      X1=PX(1)*CB(1)+H(IC)*SB(1)
      X2=PX(2)*CB(2)+HH(IC)*SB(2)
      Y1=PY(1)*SB(1)-H(IC)*CB(1)
      Y2=PY(2)*SB(2)-HH(IC)*CB(2)
      DO 330 K=1,2
      DPSIX(1,K)=X1*DBX(1,K)-Y2+DBX(2,K)+DPY(1,K)*SB(1)-DPX(2,K)*SB(2)-
     1Y2*DGYR0(K)
330   DPSIX(2,K)=-X2*DBX(1,K)+X1*DBX(2,K)+DPY(1,K)*CB(1)-DPX(2,K)*CB(2)+
     1Y1*DGYR0(K)-DGX(K)
      PSI(1)=PX(1)*SB(1)-PX(2)*SB(2)-H(1)*Y10-CY
      PSI(2)=PX(1)*CB(1)-PX(2)*CB(2)+H(1)*Y10-CX
      DET=DPSIX(1,1)*DPSIX(2,2)-DPSIX(2,1)*DPSIX(1,2)
      CBR(1)=(PSI(1)*DPSIX(2,2)-PSI(2)*DPSIX(1,2))/DET
      CBR(2)=(DPSIX(1,1)*PSI(2)-DPSIX(2,1)*PSI(1))/DET
      DO 340 K=1,2
340   X(K)=X(K)-CBR(K)*CONST
      DO 350 K=1,2
      IF(ABS(CBR(K))-TOL(K))350,350,360
360   CONTINUE
      IF(CONST-2.)1,1,350
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```
      GO TO 380
  360 CONTINUE
      WRITE(6,370)J
  370 FORMAT(11H0BALL - 370,I4)
      IQUIT=1
      RETURN
  380 ICNT=ICNT+1
      DO 390 K=1,2
      WDTH(K)=(.2387324 *ESAVE(K)*CSEP(K)*EL(K)*PX(K)*D/TSAVE(K))**.3333
     1333
  390 TRQ(K)=.375*WDTH(K)*PX(K)*ESAVE(K)/CSEP(K)
      B2B1=COS(B(2)-B(1))
      IF(ICNT-3)420,400,400
  400 IF(LOOP)410,410,510
  410 LOOP=1
      IC=1
      GO TO 5
  420 IF(IC-1)430,430,460
  430 IF(TRQ(2)*B2B1-TRQ(1))450,450,440
  440 IC1=2
      GO TO 490
  450 IC1=1
      GO TO 490
  460 IF(TRQ(1)*B2B1-TRQ(2))480,480,470
  470 IC1=1
      GO TO 490
  480 IC1=2
  490 IF(IC1-IC)500,510,500
  500 IC=IC1
      GO TO 5
  510 DDELA(2,1)=SB(2)
      DDELA(2,2)=CB(2)
      DBA(2,1)=CB(2)**2/(A2-X(2))
      DBA(2,2)=-(A1-X(1))*(CB(2)/(A2-X(2)))**2
      DGA(2,1)=-D*DBA(2,1)*SB(2)/E2
      DGA(2,2)=-D*DBA(2,2)*SB(2)/E2
      DO 530 K=1,2
      DALA=(Y1*Y2*DBA(IC,K)-Y3*(-2.*Y3*DBA(IC,K)+C(IC)*DGA(IC,K)))*Y4
      V1(K)=GG(1)*SBAL(2)*(DGA(2,K)-DALA)
      V2(K)=GG(2)*SBAL(1)*(DGA(1,K)-DALA)
      DOMBA=.5*(D*(-GG(1)*DGA(2,K)+GG(2)*DGA(1,K))-GG(1)*GG(2)*(-V1(K)+
     1CBAL(2)*DGA(1,K)-V2(K)-CBAL(1)*DGA(2,K)))
      DOMEA=(DN*(RPM(1)*(-V1(K)+CBAL(2)*DGA(1,K))-RPM(2)*(V2(K)+CBAL(1)*
     1DGA(2,K)))-Y6*(-V1(K)+CBAL(2)*DGA(1,K)-V2(K)-CBAL(1)*DGA(2,K)))/DN
     2**2
      DCA=XMASS*E2*REVE*DOMBA
      DGYA(K)=XMN*((REVB*DOMEA+REVE*DOMBA)*SAL+REVB*REVE*CAL*DALA)
      DO 520 L=1,2
  520 DBA(L,K)=1.5*SB(L)*DDELA(1,K)+DEL(L)*SQRT(DEL(L))*DXKDG(L)*DGA(L,K
     1)
      DDSLA(1)=Y1*DBA(1,K)-Y2*DBA(2,K)+DBA(1,K)*SB(1)-DBA(2,K)*SB(2)-Y7*
     1DGYA(K)
```


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```
      DPSIA(2)=-A3*DBA(1,K)+A4*DBA(2,K)+DPA(1,K)*CB(1)-DPA(2,K)*CB(2)+Y8
     1*DGYA(K)=DCA
      DXA(1)=(-DPSIA(1)*DPSIX(2,2)+DPSIA(2)*DPSIX(1,2))/DET
      DXA(2)=(-DPSIA(2)*DPSIX(1,1)+DPSIA(1)*DPSIX(2,1))/DET
      DPA(2,K)=DBA(2,K)+DBX(2,1)*DXA(1)+DBX(2,2)*DXA(2)
      DDELA(2,K)=DDELA(2,K)+DDELX(2,1)*DXA(1)+DDELY(2,2)*DXA(2)
      DPA(2,K)=1.5*SQ(2)*DDELA(2,K)+DEL(2)*SQRT(DEL(2))*DXKDG(2)*(DGA(2,
     1K)+DGX(2,1)*DXA(1)+DGY(2,2)*DXA(2))
  530 DGYA(K)=DGYA(K)+DGYRO(1)*DXA(1)+DGYRO(2)*DXA(2)
      Y1=PX(2)*CB(2)-HH(IC)*CB(2)
      Y2=PX(2)*SB(2)+HH(IC)*SB(2)
      Y3=R*Y1+HH(IC)*F(2)*D
      XF1(1)=XF1(1)+Y1
      XF1(2)=XF1(2)+Y2*CPH                                              BALL 257
      XF1(3)=XF1(3)+Y2*SPH                                              BALL 258
      XF1(4)=XF1(4)+Y3*SPH                                              BALL 259
      XF1(5)=XF1(5)+Y3*CPH                                               13L- 60
      DO 540 K=1,5
      C1=DBA(2,1)*DAD(1,K)+DBA(2,2)*DAD(2,K)
      C2=DPA(2,1)*DAD(1,K)+DPA(2,2)*DAD(2,K)
      C3=DGYA(1)*DAD(1,K)+DGYA(2)*DAD(2,K)
      C4    =A2*C1+C2*SB(2)-H2(IC)*C3*CB(2)*2./D
      C5    =-A4*C1+C2*CB(2)+H2(IC)*C3*SB(2)*2./D
      C6=R*C4+HH(IC)*F(2)*C3*2.
      DTV(1,K)=DTV(1,K)+C4
      DTV(2,K)=DTV(2,K)+C5*CPH                                          BALL 269
      DTV(3,K)=DTV(3,K)+C5*SPH                                          BALL 270
      DTV(4,K)=DTV(4,K)+C6*SPH                                          BALL 271
  540 DTV(5,K)=DTV(5,K)+C6*CPH                                          BALL 272
      DO 550 K=1,2
      P(K,J)=PX(K)
      SMAJ(K,J)=WDTH(K)*2./CSEP(K)
      SMIN(K,J)=2.*WDTH(K)
      XDEL(K,J)=DEL(K)
  550 PT(K,J)=Q(K)*57.295780
      YCA(J)=CY
      XGYRO(J)=GYRO
      XAL(J)=ALX*57.295780
      ICTL(J)=IC
      DDEV(J)=RDEVE/.1047198
      QDEV(J)=RDEVQ/.1047198
      RETURN
      END
      SUBROUTINE ELIPIN(TERM1,TERM2,EE,EK,CSE2,IER)                     4046
      CTAU=TERM1/TERM2                                                  4047
      SUE=.0999                                                         4048
      IER=0                                                             4049
      DO 40 I=1,30                                                      4050
      SUES=SUE*SUE                                                      4051
      CSE2=1.-SUES                                                      4052
      IF (CSE2)10,10,30                                                 4053
   10 WRITE (6,20)                                                      4054
```

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```
20  FORMAT(12HDELTPIN - 10)
    IER=1
    RETURN
30  CTE2=CSE2/SNES
    XLG=ALOG(1./CSE2)
    FE=1.+CSE2*(.44479204?+CSE2*(.085099193+CSE2*.040905094))+CSE2*(.2
   149697949+CSE2*(.08150?240+CSE2*.01382999?))*XLG
    FK=1.3862?436 +CSE2*(.097?32891+CSE2*(.?54544409+.032?24666*CSE2))
   1+(.5+CSE2*(.124750742+CSE2*(.060118519+.010944912*CSE2)))*XLG
    EUK=FK/EE
    COR=(1.-CTAU-2.*(EUK-1.)*CTE2)*SNES*SNF/((8.-4.*SNES)*EUK-2.*CSE2*
   1EUK**2-6.)
    SNE=SNE-COR
    IF (ABS(COR)-5.E-7)60,40,40
40  CONTINUE
    WRITE (6,50)
50  FORMAT(12HDELTPIN - 40)
    IER=1
60  RETURN
    END
    SUBROUTINE SIMULT(A,N,B,X,KX)
    DIMENSION A(3,3),ROW(3),B(3),X(3) ,KOL(3)
5000 KOL(1)=1
5001 DO 5002  IROW=2,N
5002 KOL(IROW)=KOL(IROW-1)+1
5004 DO 5020  COUNT=1,N
    LARGST=N-COUNT+1
    IERASE=KOL(1)
    JCOL=1
5005 IF  (N-COUNT)  5035,5014,5006
5006    AMPY =ABS ( A(1,1) )
5007 DO 5010  IROW=2,LARGST
5008    IF ( AMPY - ABS ( A(IROW,1))) 5009, 5010, 5010
5009 JCOL=IROW
        AMPY = ABS ( A(IROW,1) )
    IERASE=KOL(IROW)
5010 CONTINUE
5011 IF (KOL(1)-IERASE)  5012,5014,5012
5012 KOL(JCOL)=KOL(1)
    KOL(1)=IERASE
5014 IF (A(JCOL,1))5015,5035,5015
5015 AMPY=A(JCOL,1)
5017 DO 5018  IROW=2,N
    ROW(IROW-1)=A(JCOL,IROW)/AMPY
5018 A(JCOL,IROW-1)=A(1,IROW-1)
    ROW(N)=1./AMPY
    A(JCOL,N)=A(1,N)
5019 DO 5020  IROW=2,N
    AMPY=A(IROW,1)
5020 DO 5021  JCOL=2,N
5021 A(IROW-1,JCOL-1)=A(IROW,JCOL)-AMPY*ROW(JCOL-1)
5022 A(IROW-1, )=-AMPY*ROW( )
```

```
5023 DO 5024   JCOL=1,N                                                    2048
     KOL(JCOL)=KOL(JCOL+1)                                                2049
5024 A(N,JCOL)=ROW(JCOL)                                                  2050
5025 KOL(N)=IERASE                                                        2051
5026 DO 5033   KOUNT=1,N                                                  2052
5027 IF (KOL(KOUNT)-KOUNT)  5035,5034,5028                                2053
5028 DO 5032   IROW=KOUNT,N                                               2054
5029 IF (KOL(IROW)-KOUNT)  5035,5030,5032                                 2055
5030 DO 5031   JCOL=1,N                                                   2056
     ROW(1)=A(JCOL,IROW)                                                  2057
     A(JCOL,IROW)=A(JCOL,KOUNT)                                           2058
5031 A(JCOL,KOUNT)=ROW(1)                                                 2059
     IERASE=KOL(KOUNT)                                                    2060
     KOL(KOUNT)=KOL(IROW)                                                 2061
     KOL(IROW)=IERASE                                                     2062
     GO TO 5033                                                           2063
5032 CONTINUE                                                             2064
     GO TO 5035
5034 CONTINUE
997  IF(KX.NE.3)GO TO 998                                                 2067
     KX=0                                                                 2068
     RETURN                                                               2069
998   DO 5042 IROW=1,N
5040    X(IROW) =0.0                                                      2072
5041    DO 5042  JKOL=1,N
5042    X(IROW) = X(IROW)+A(IROW,JKOL)   *   B(JKOL)                      2073
     KX=0
                 RETURN                                                   2074
5035 KX=1                                                                 2075
     RETURN                                                               2076
     END                                                                  2077
```


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APPENDIX B

BEARING STIFFNESS DESIGN CHARTS

PURE RADIAL LOAD

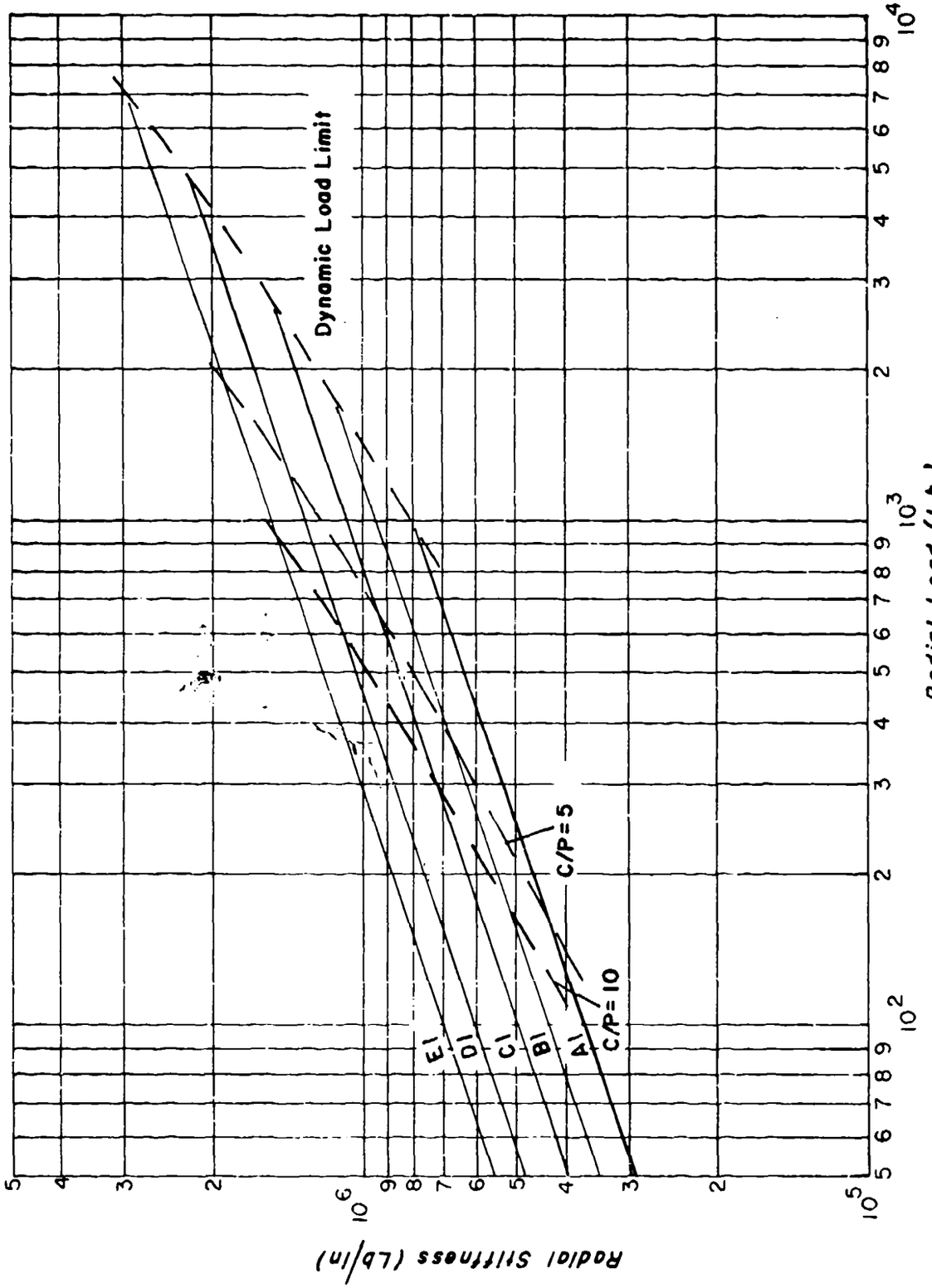

Fig. B-1 Radial Stiffness for Deep Groove Ball Bearing, Pure Radial Load

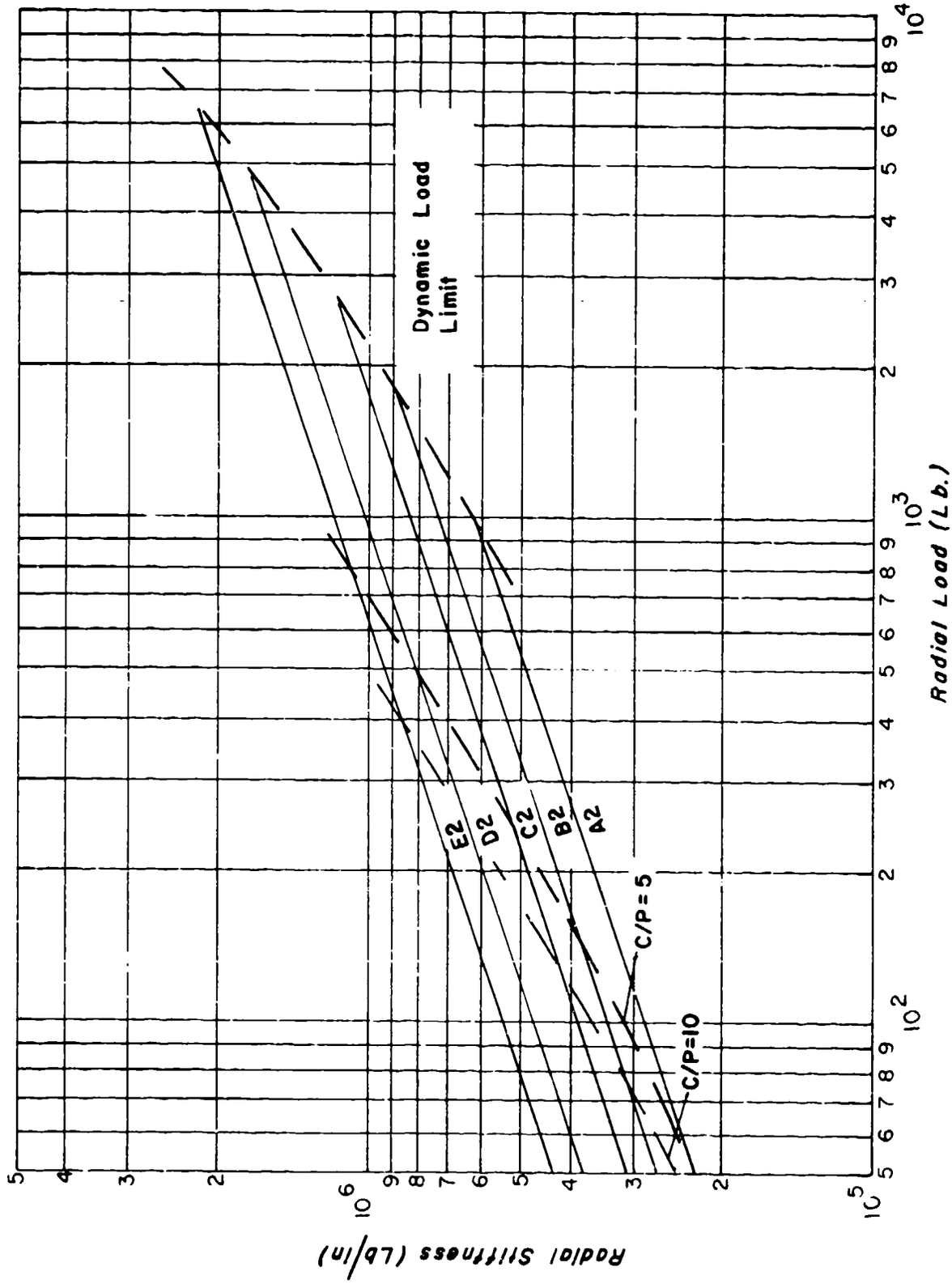

Fig. B-2 Radial Stiffness for Deep Groove Ball Bearings, Pure Radial Load

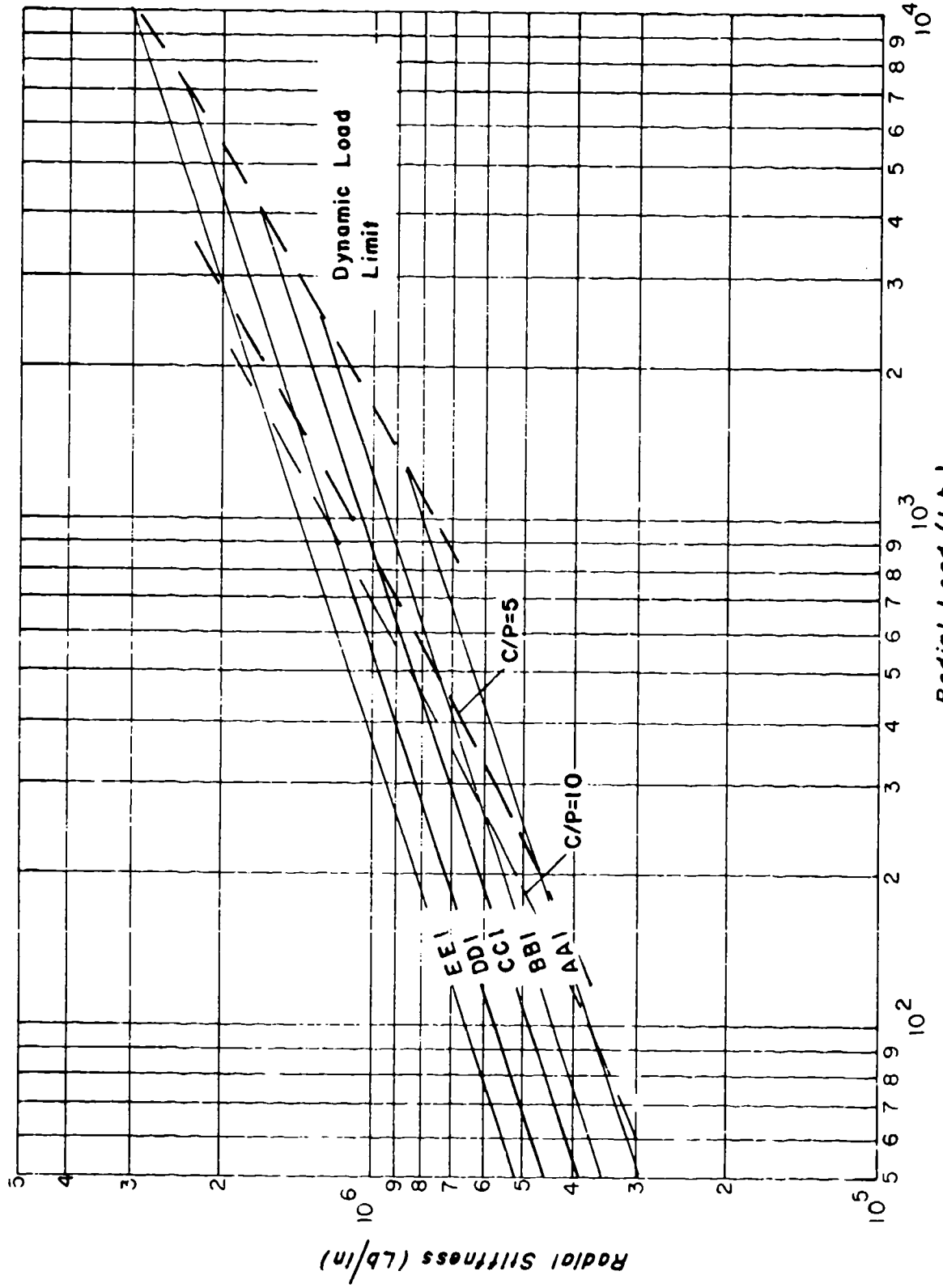

Fig. B-3 Radial Stiffness for Deep Groove Ball Bearings, Pure Radial Load

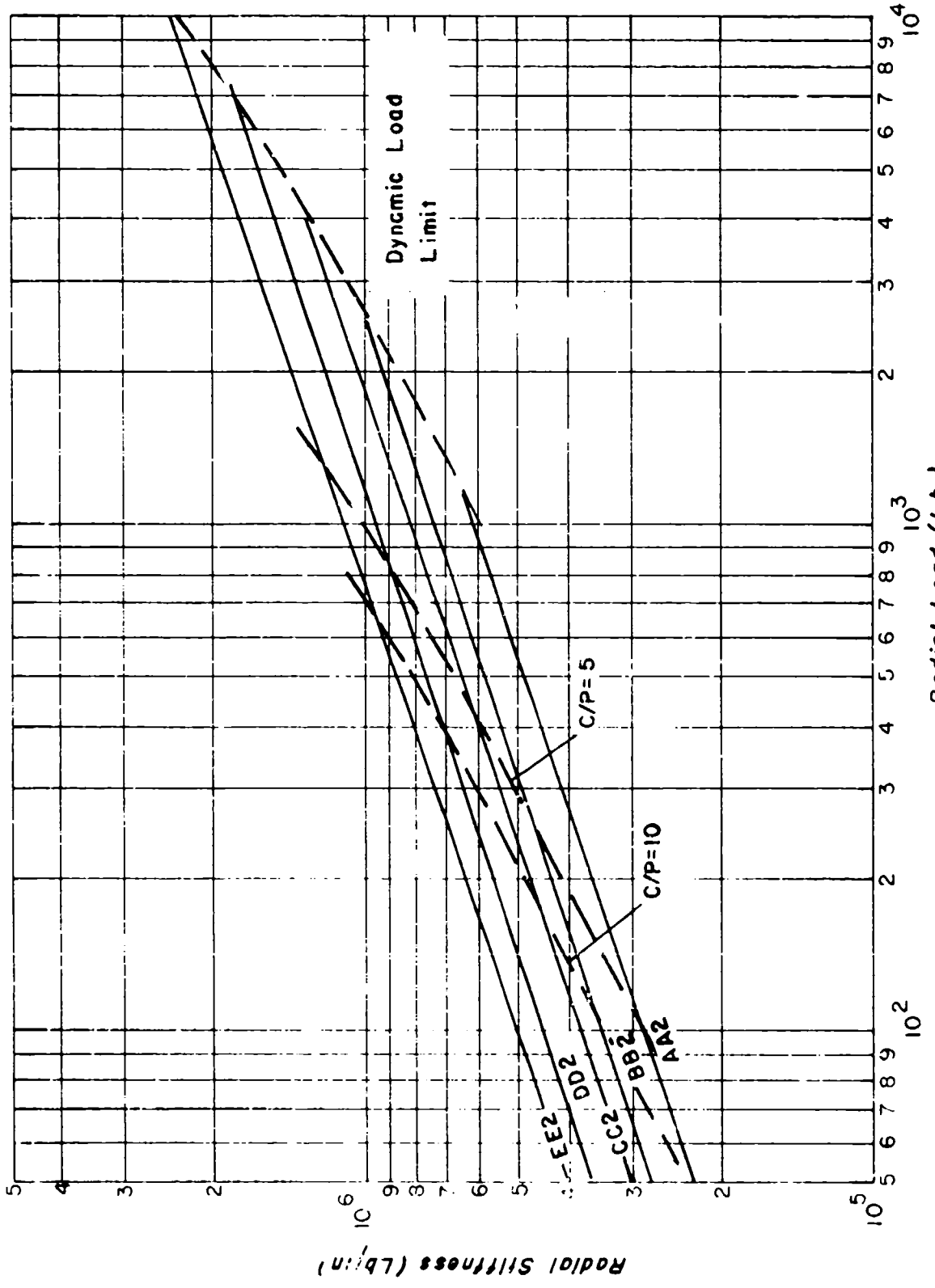

Fig. B-4 Radial Stiffness for Deep Groove Ball Bearings, Pure Radial Load

APPENDIX C

BEARING STIFFNESS DESIGN CHARTS

PURE THRUST LOAD

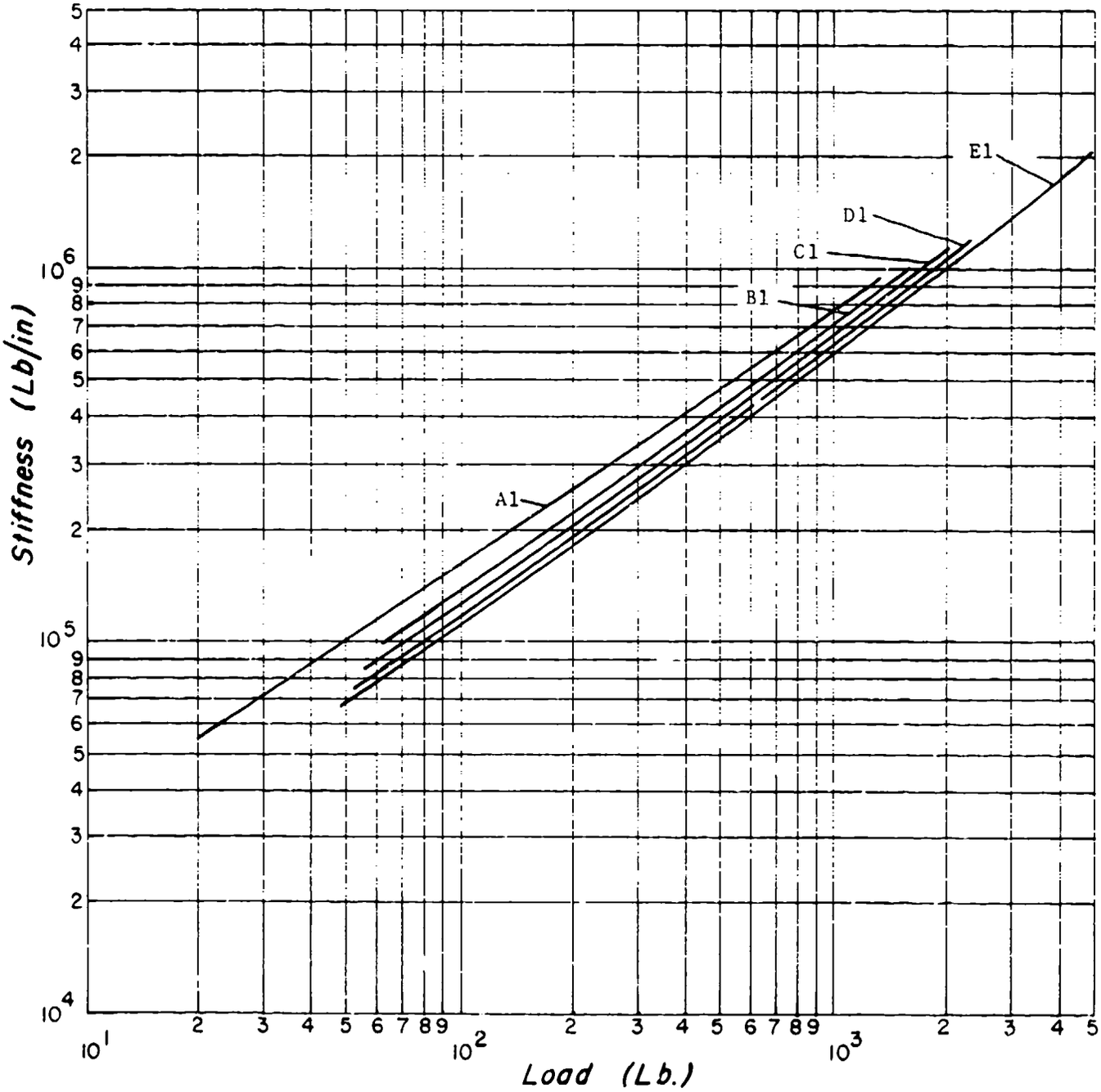

Fig. C-1 Axial Stiffness versus Axial Load, No Radial Load

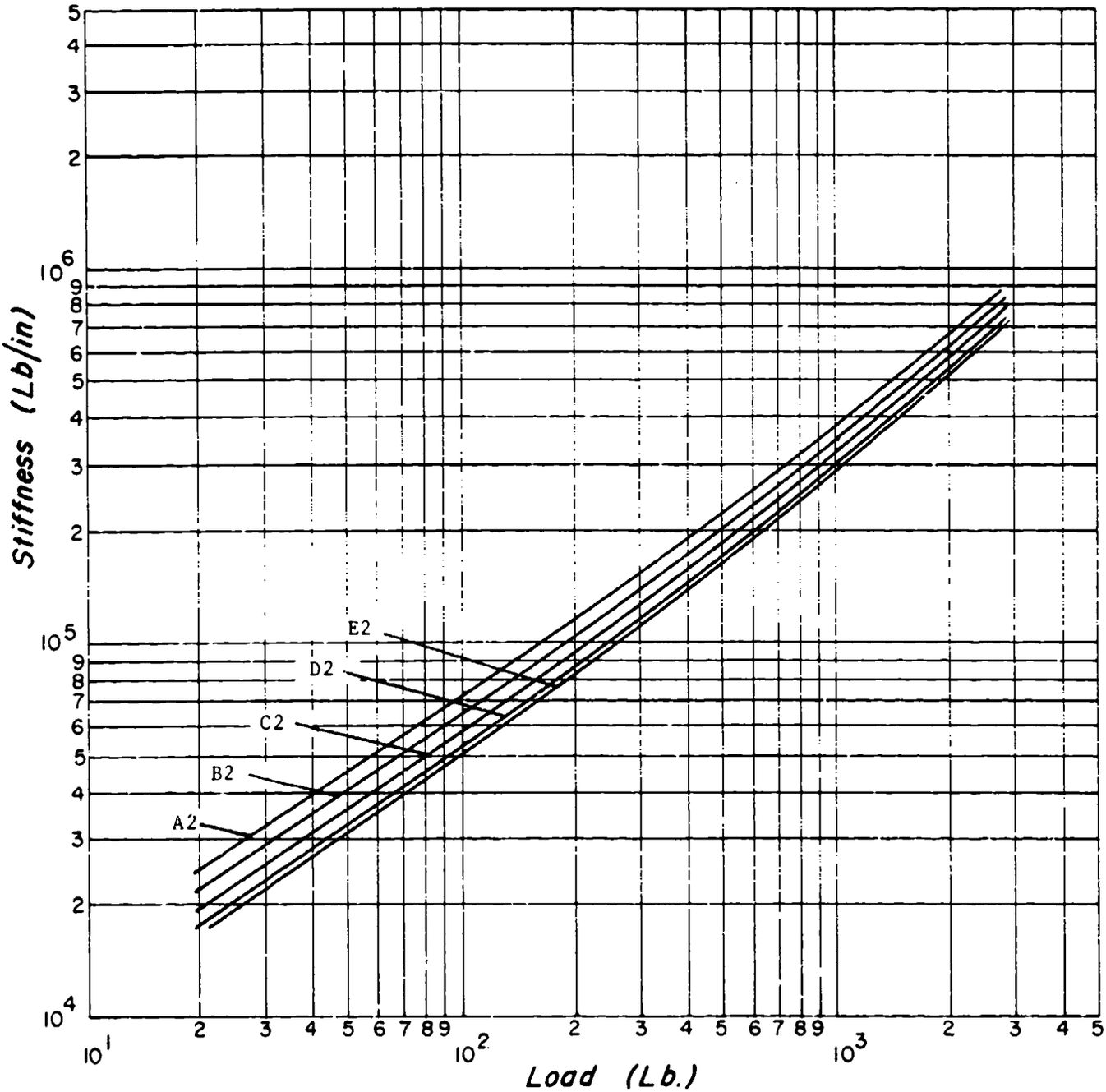

Fig. C-2 Axial Stiffness versus Axial Load, No Radial Load

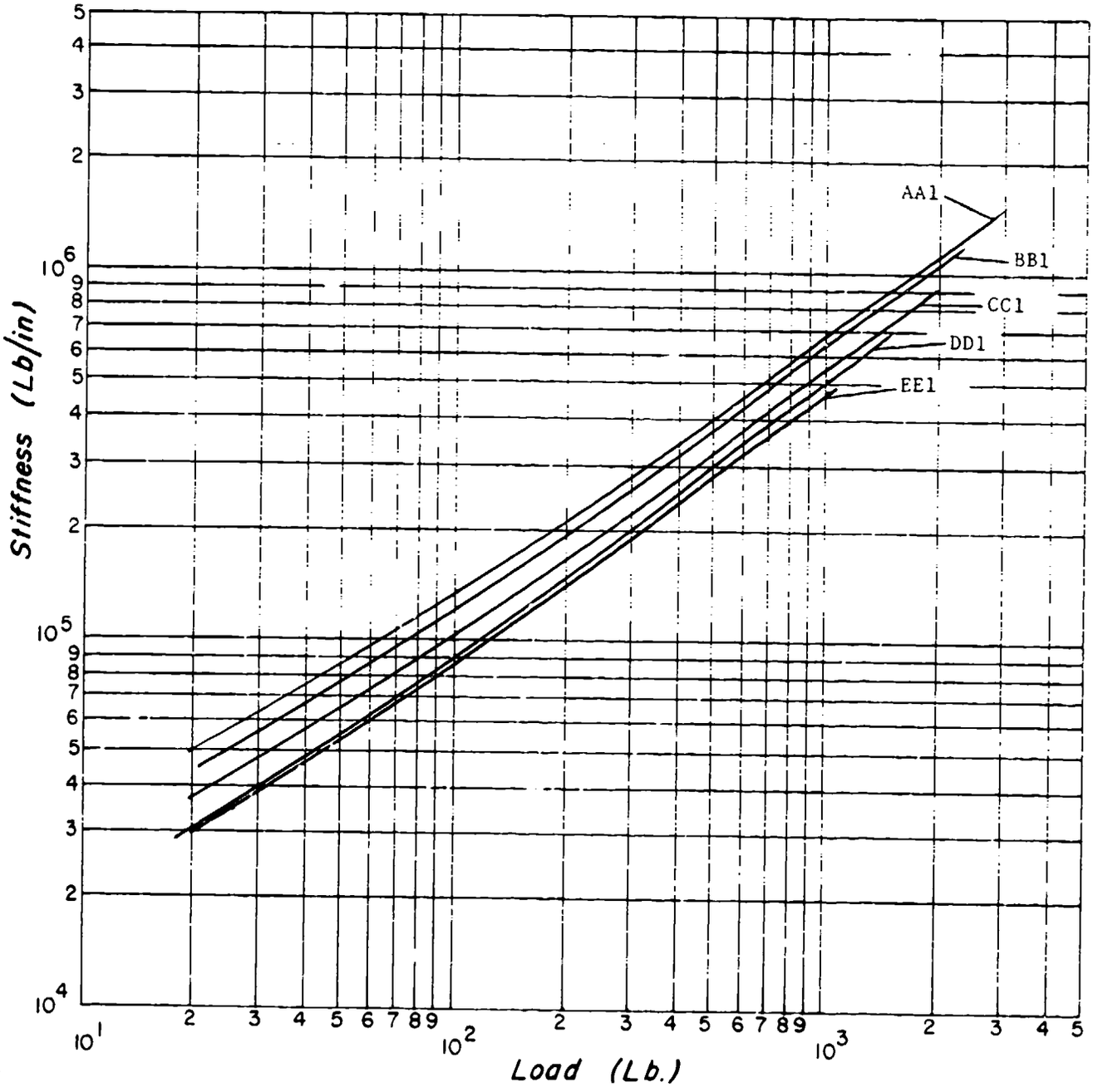

Fig. C-3 Axial Stiffness versus Axial Load, No Radial Load

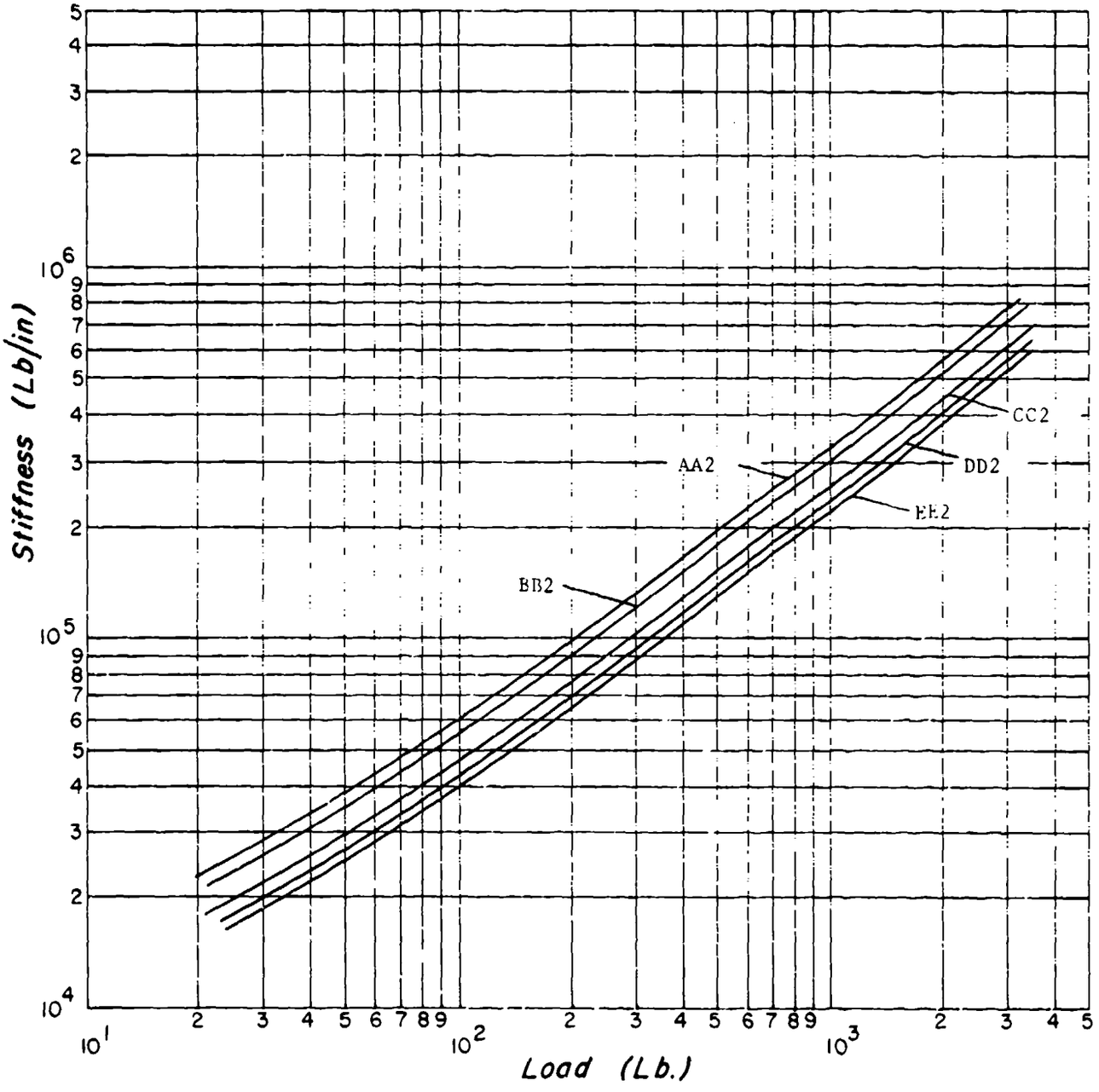

Fig. C-4 Axial Stiffness versus Axial Load, No Radial Load

APPENDIX D

BEARING STIFFNESS DESIGN CHARTS

ANGULAR CONTACT BEARINGS WITH PRELOAD

$\beta = 25^{\circ}$

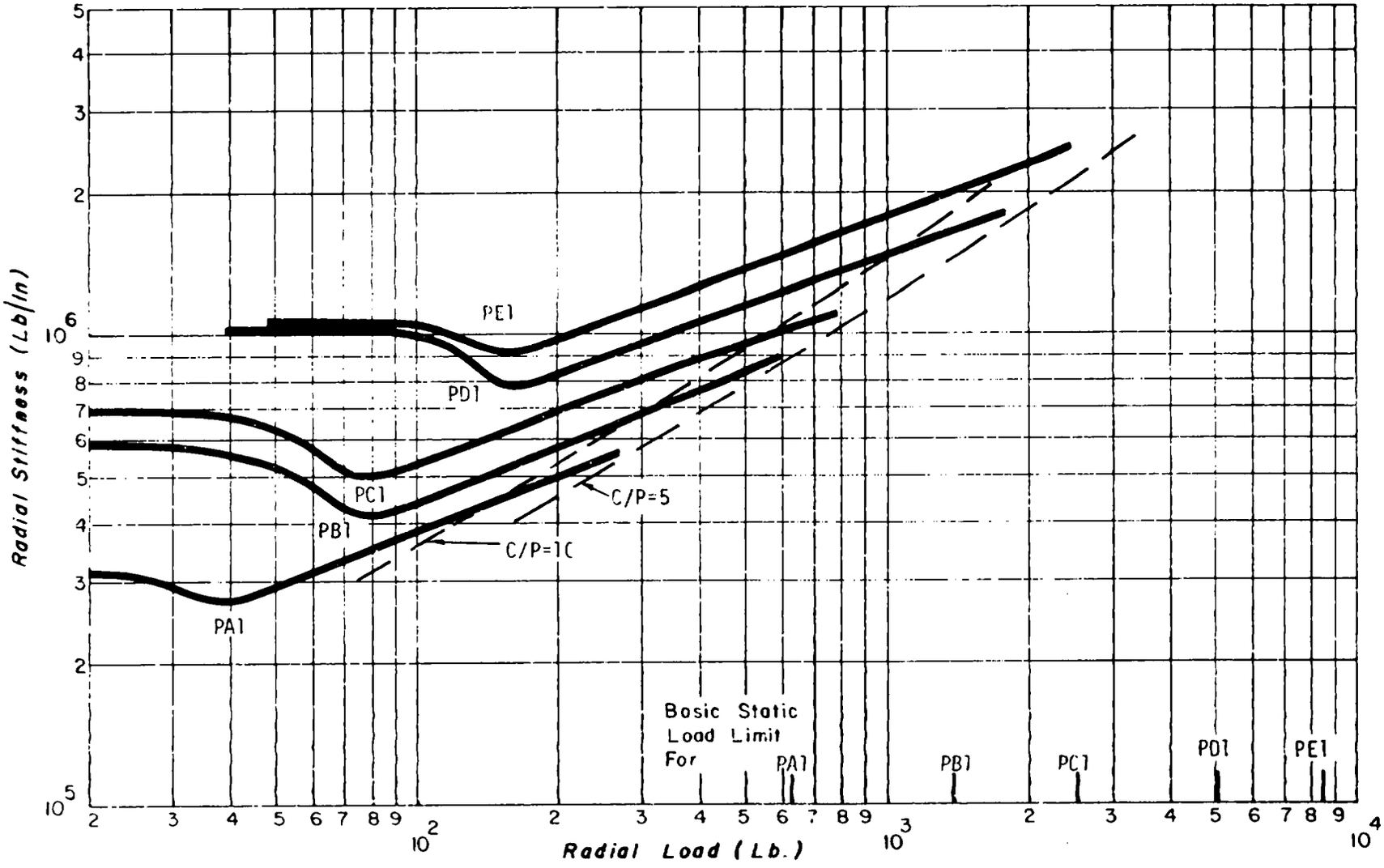

Fig. D-1 Radial Stiffness for Angular Contact Bearing, Preload -- Selected Light, $\beta = 25^{\circ}$

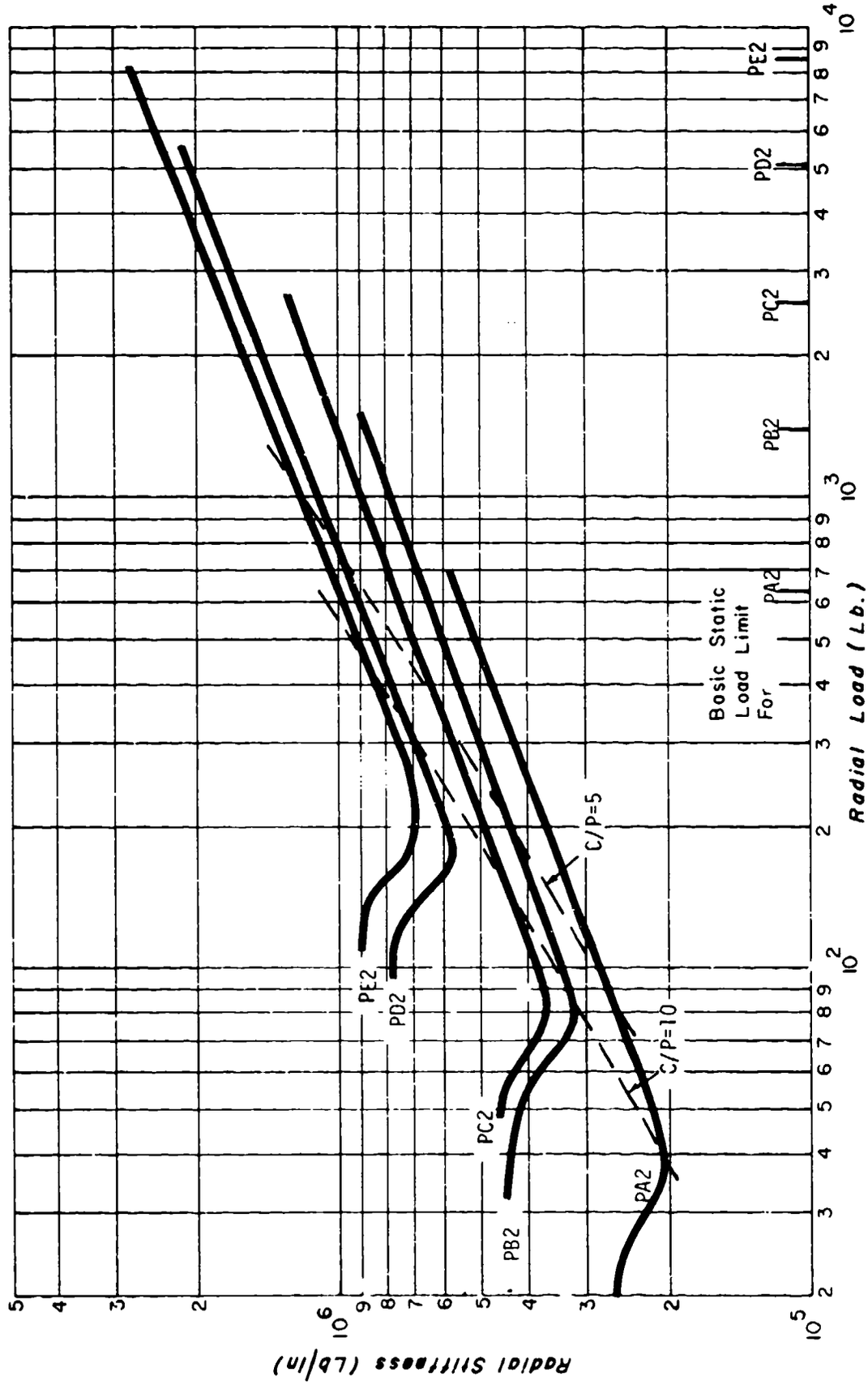

Fig. D-2 Radial Stiffness for Angular Contact Bearing, Preload - Selected Light, $\beta = 25^{\circ}$

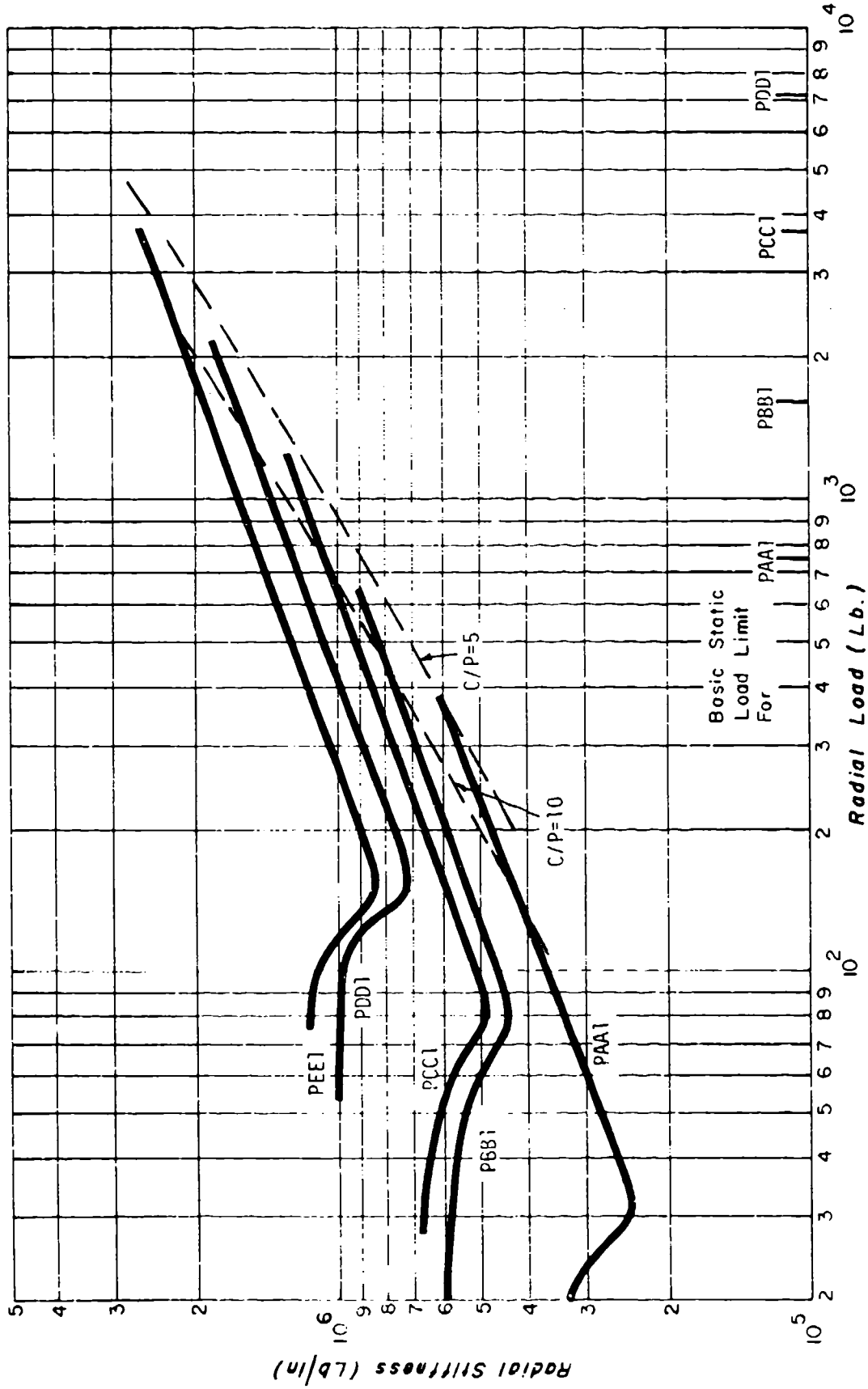

Fig. D-3 Radial Stiffness for Angular Contact Bearing, Preload -- Selected Light, $\phi = 25°$

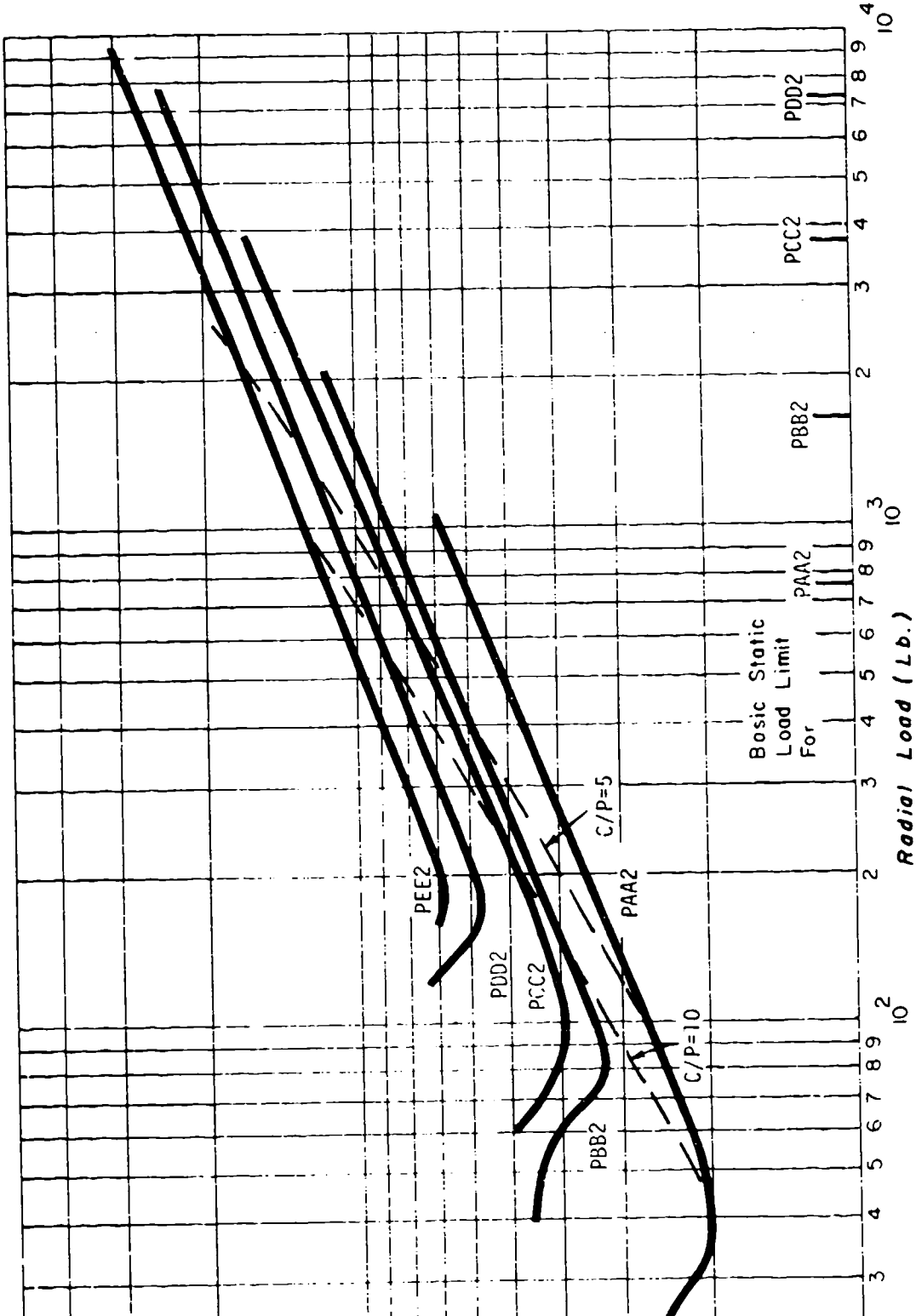

Fig. B-4 Radial Stiffness for Angular Contact Bearings, Preload -- Selected Light, F = 250

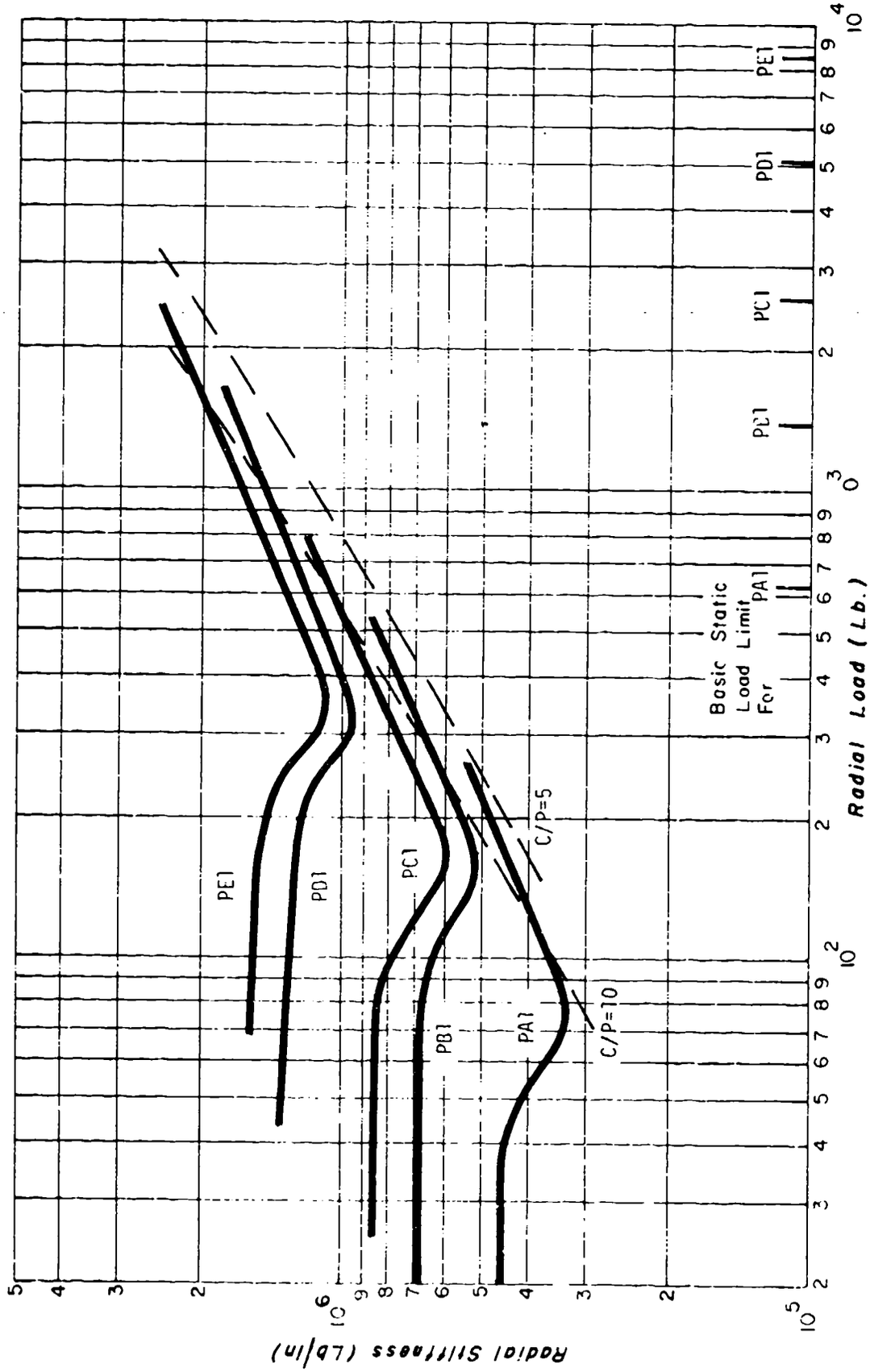

Fig. D-5 Radial Stiffness for Angular Contact Bearing, Preload -- Moderate, $\beta = 25^{\circ}$

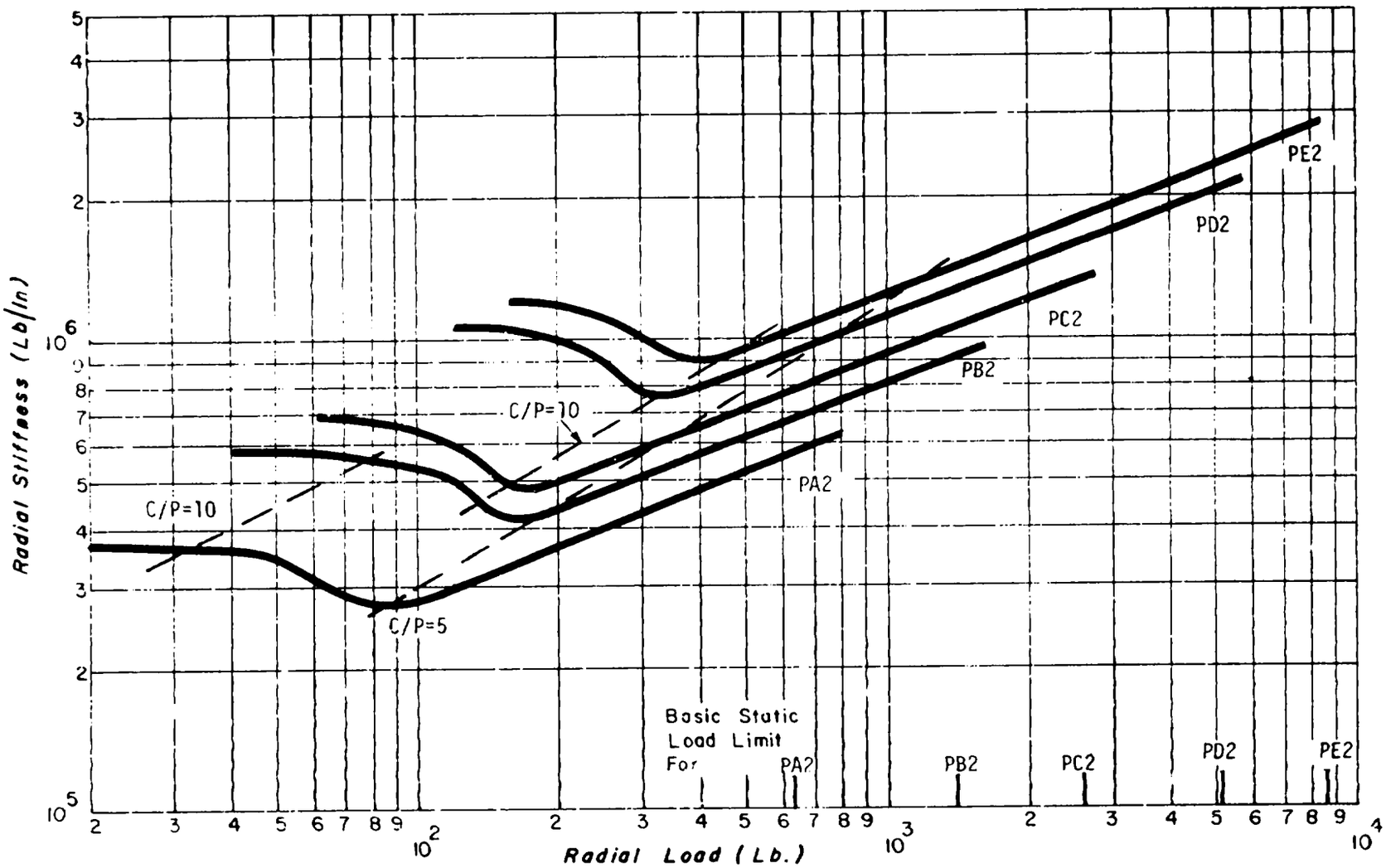

Fig. D-6 Radial Stiffness for Angular Contact Bearing, Preload -- Moderate, $\theta = 25^{\circ}$

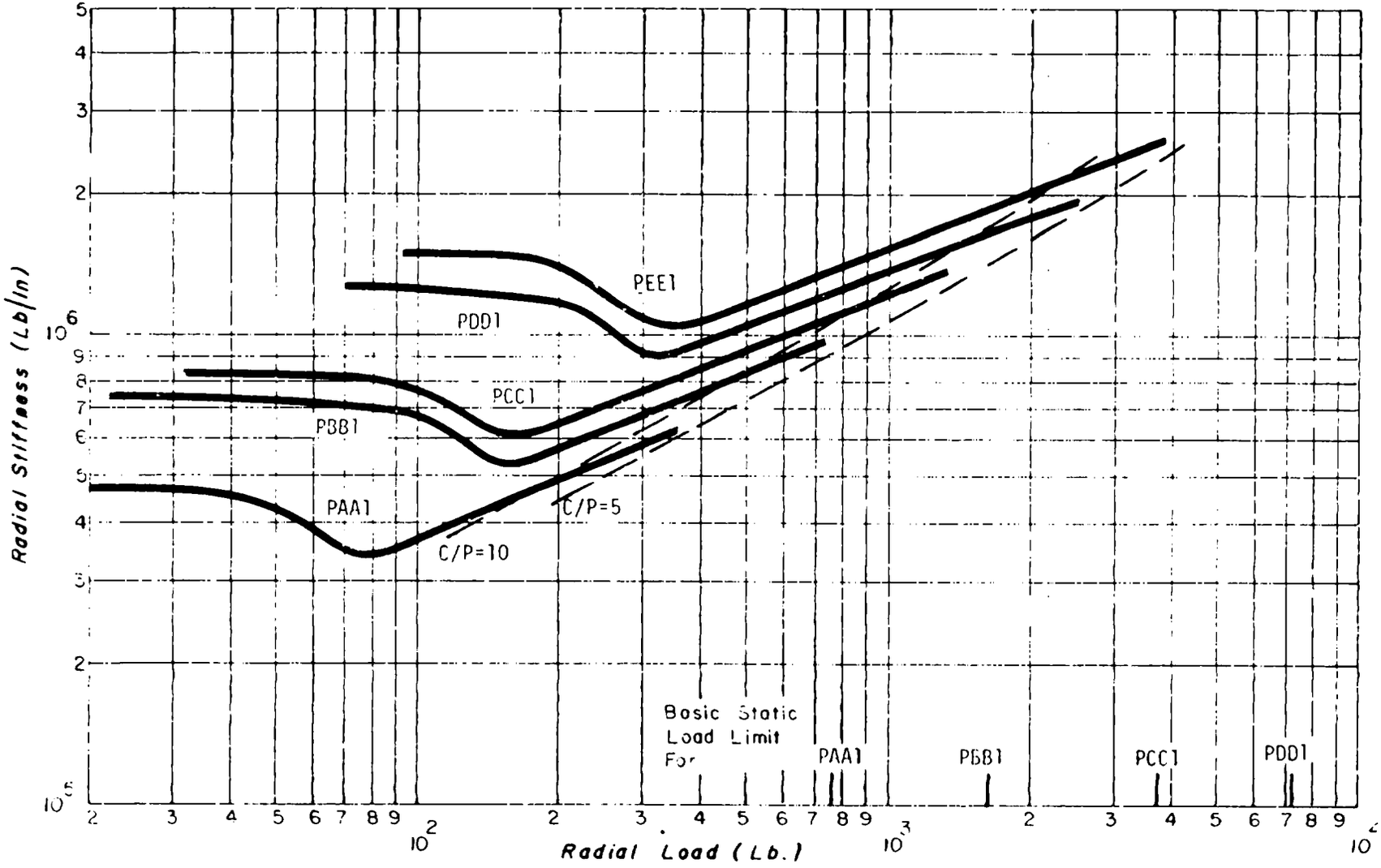

Fig. D-7 Radial Stiffness for Angular Contact Bearing, Preload -- Moderate, $\beta = 25^{\circ}$

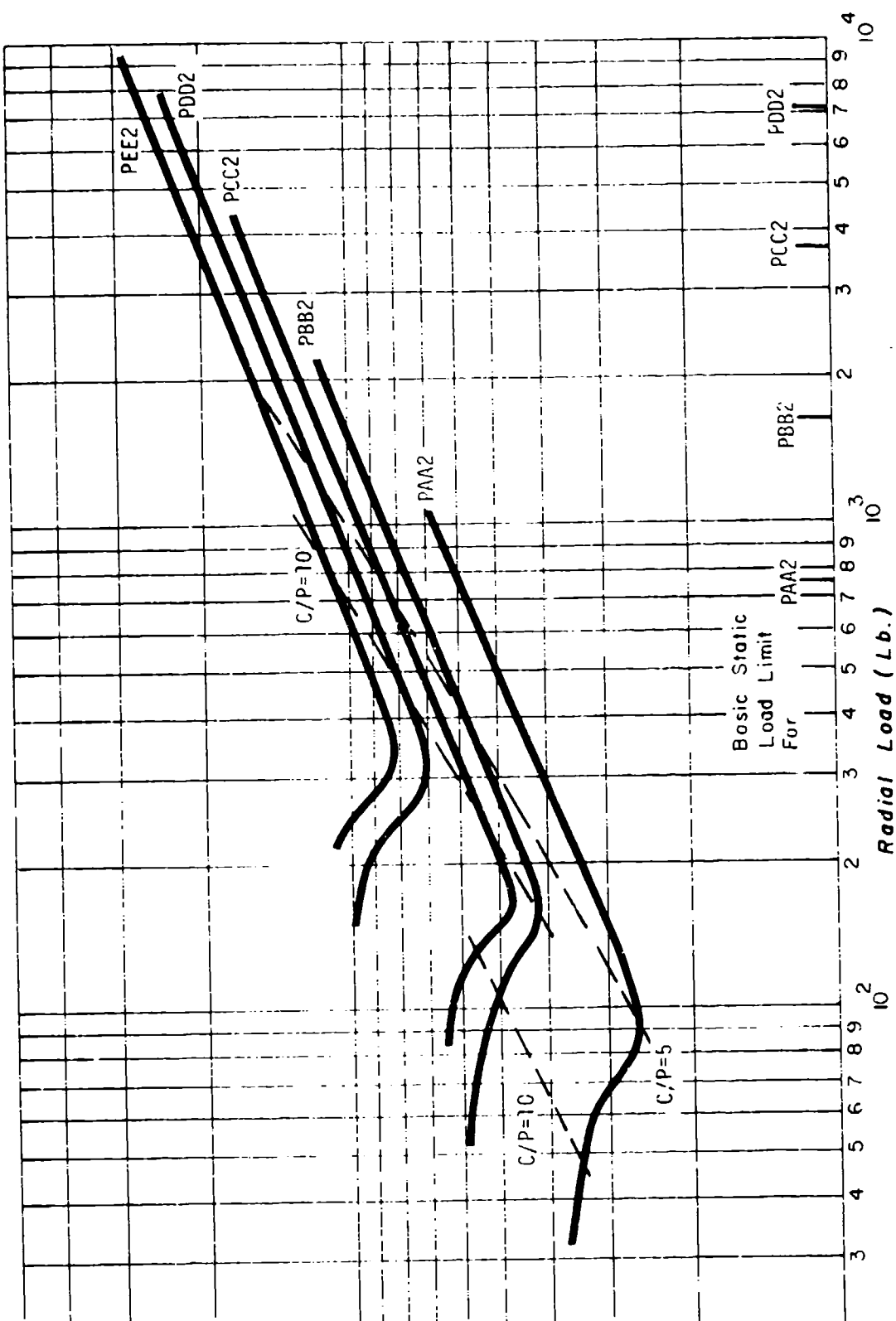

Fig. D-8 Radial Stiffness for Angular Contact Bearing, Preload -- Moderate, $\beta = 25°$

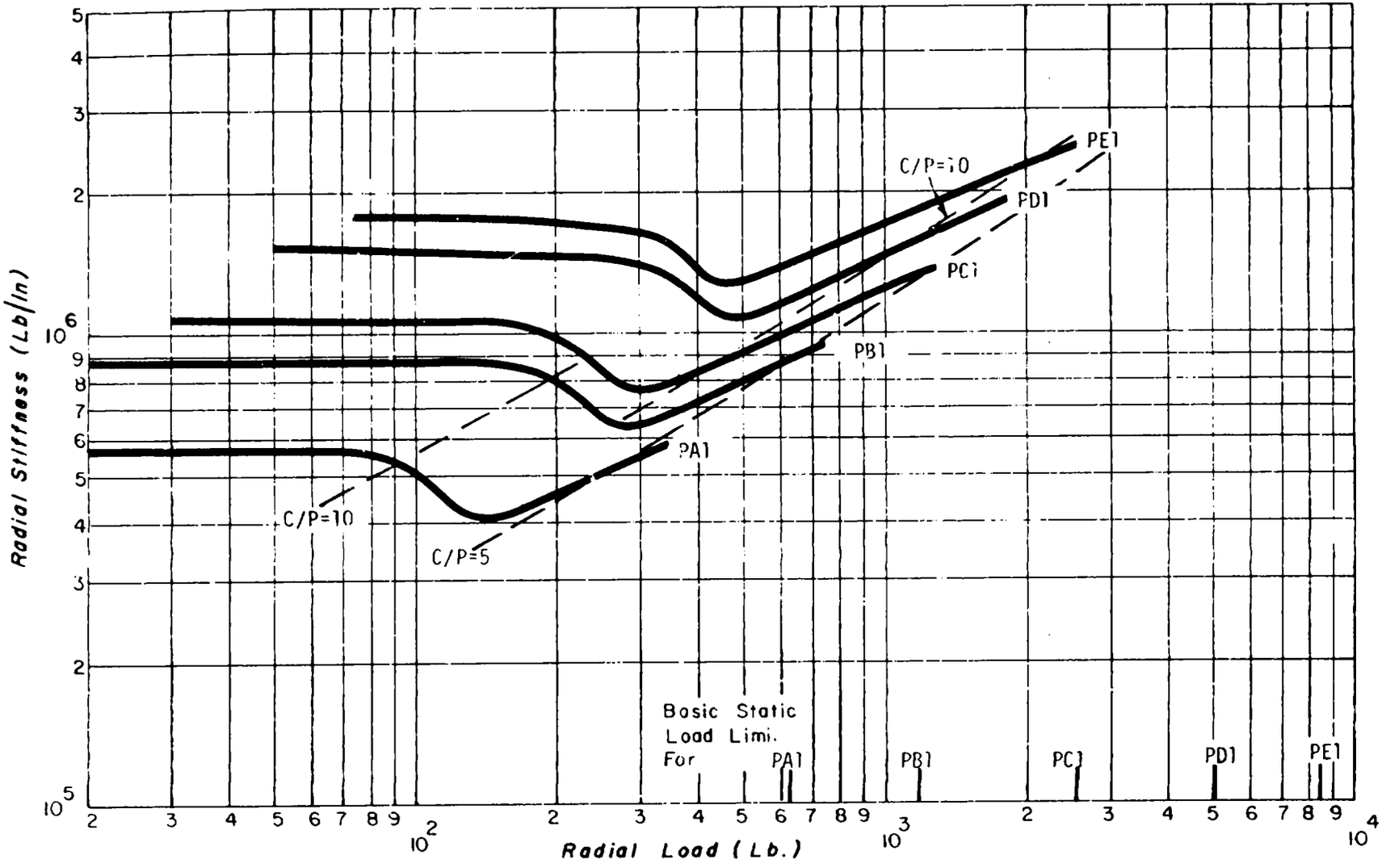

Fig. D-9 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\beta = 25^{\circ}$

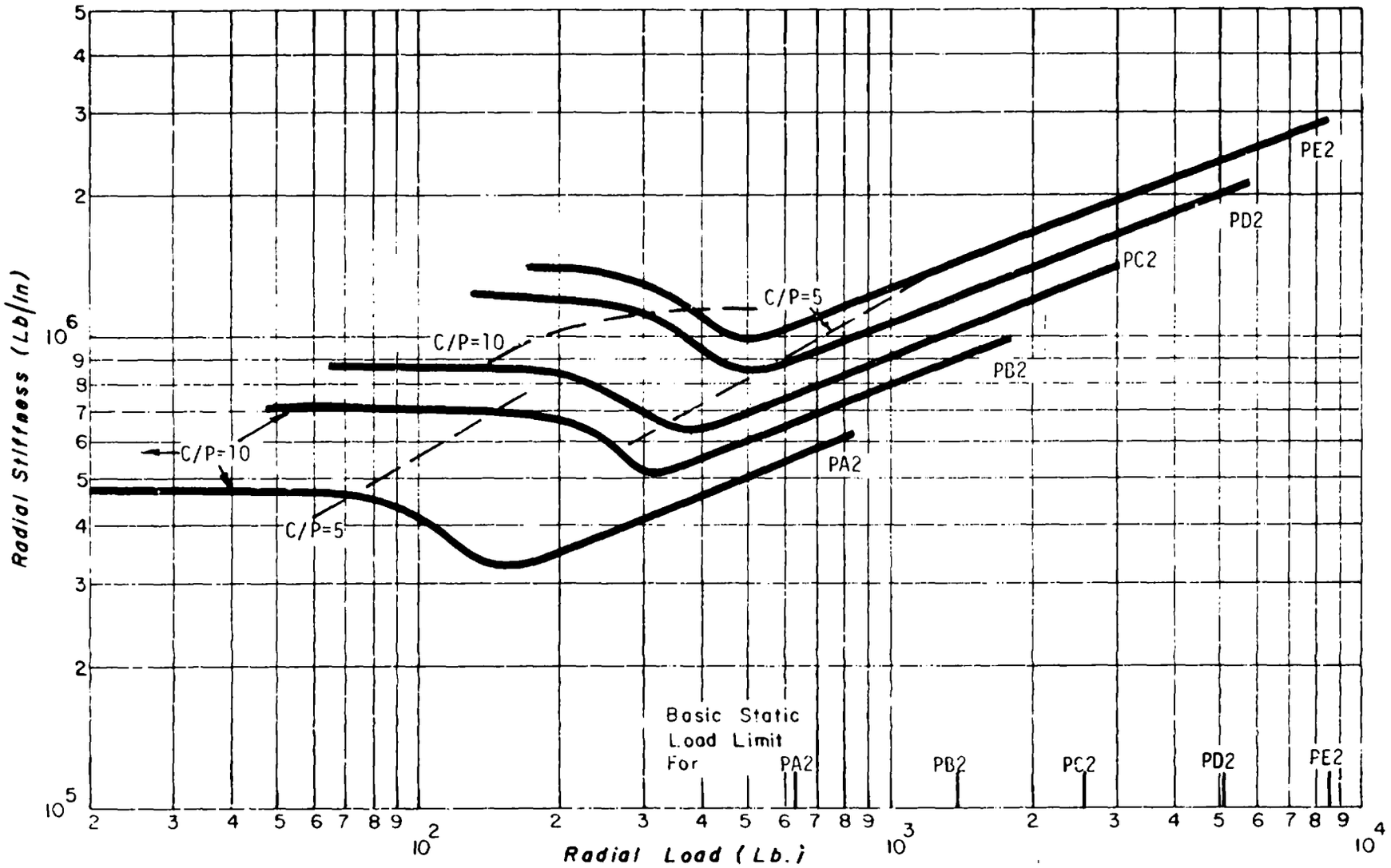

Fig. 5-10 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\beta = 25^{\circ}$

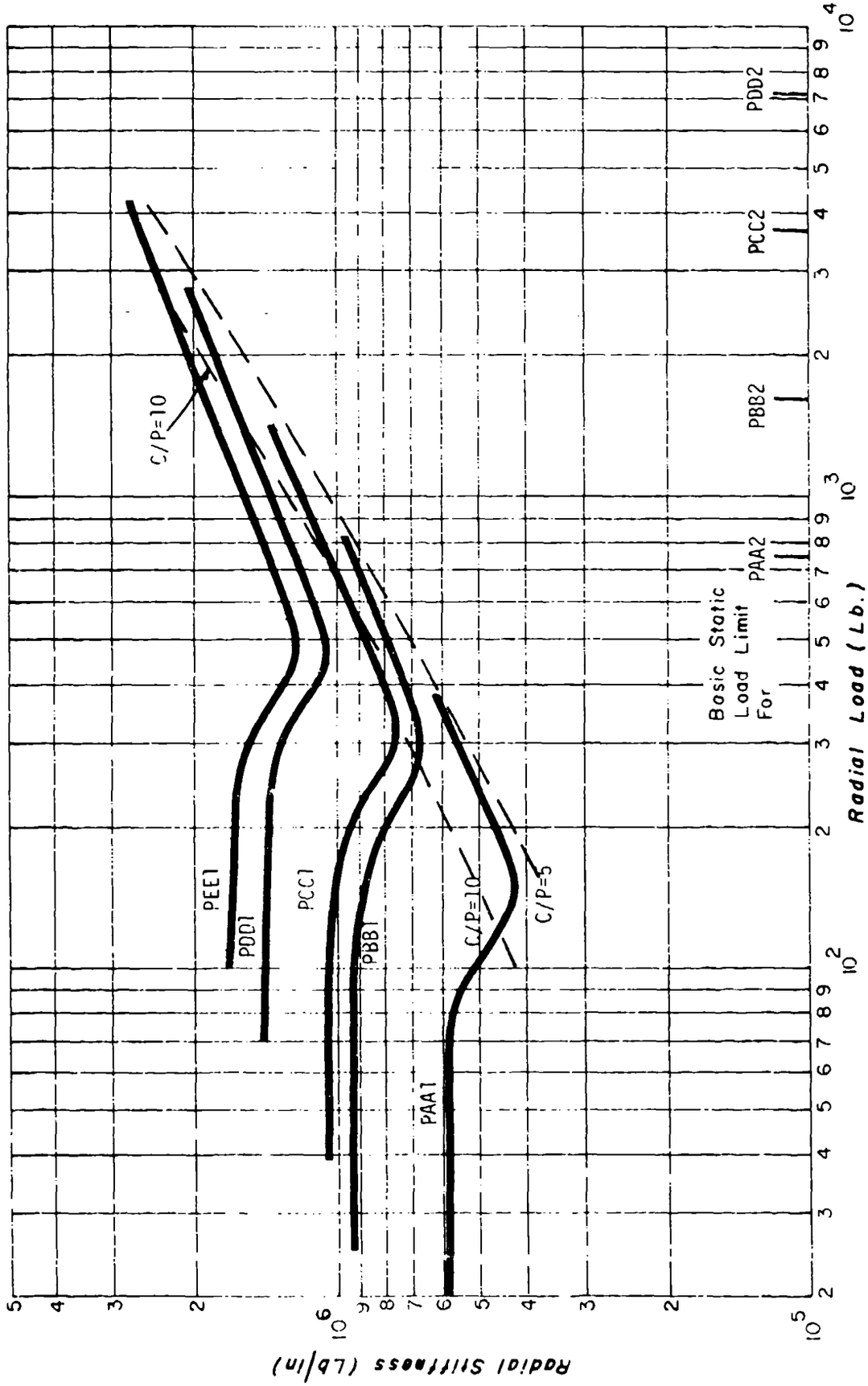

Fig. D-21 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\phi = 25°$

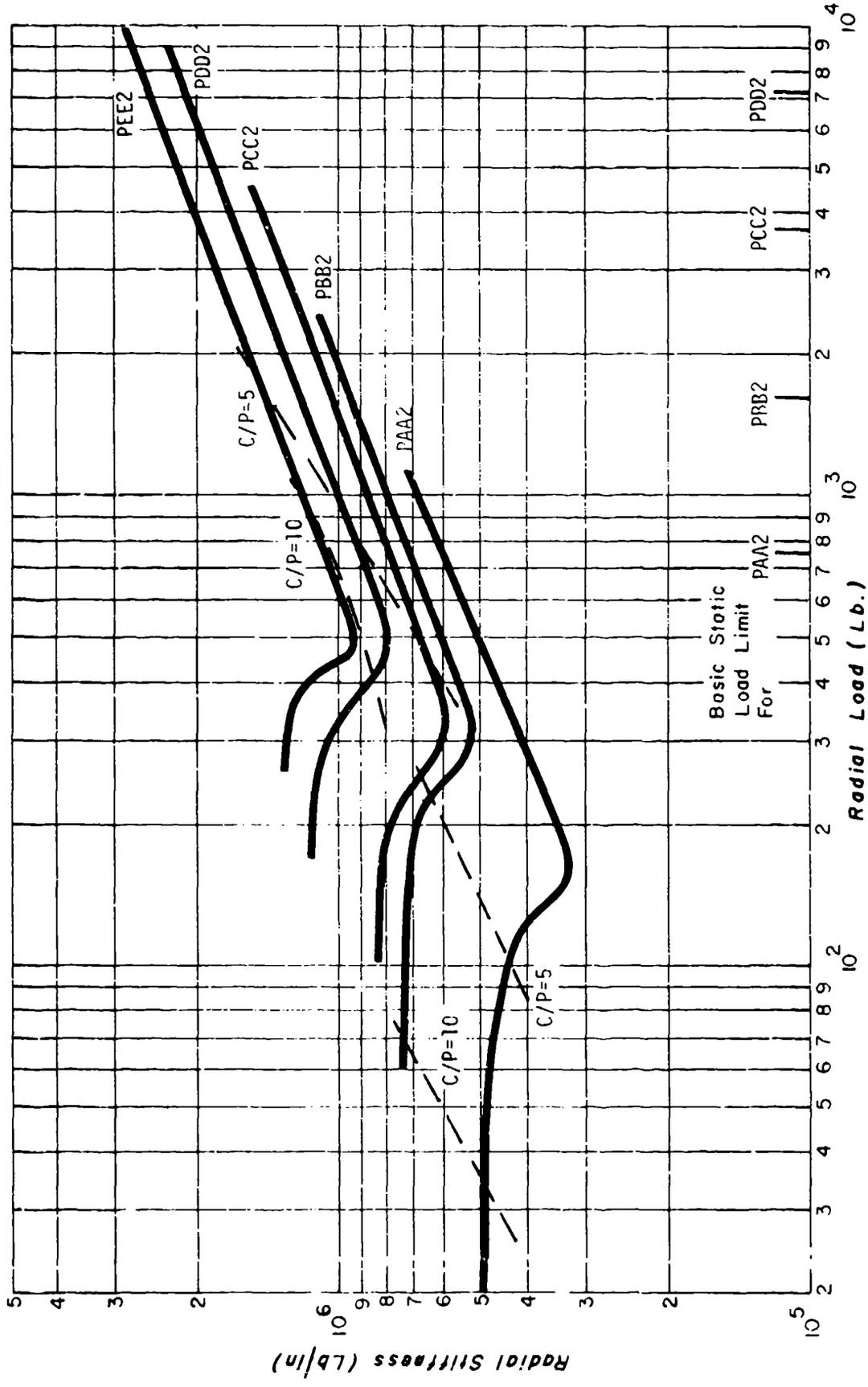

Fig. D-12 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\beta = 25°$

APPENDIX E

BEARING STIFFNESS DESIGN CHARTS
ANGULAR CONTACT BEARING WITH PRE-LOAD

$\beta = 15^{\circ}$

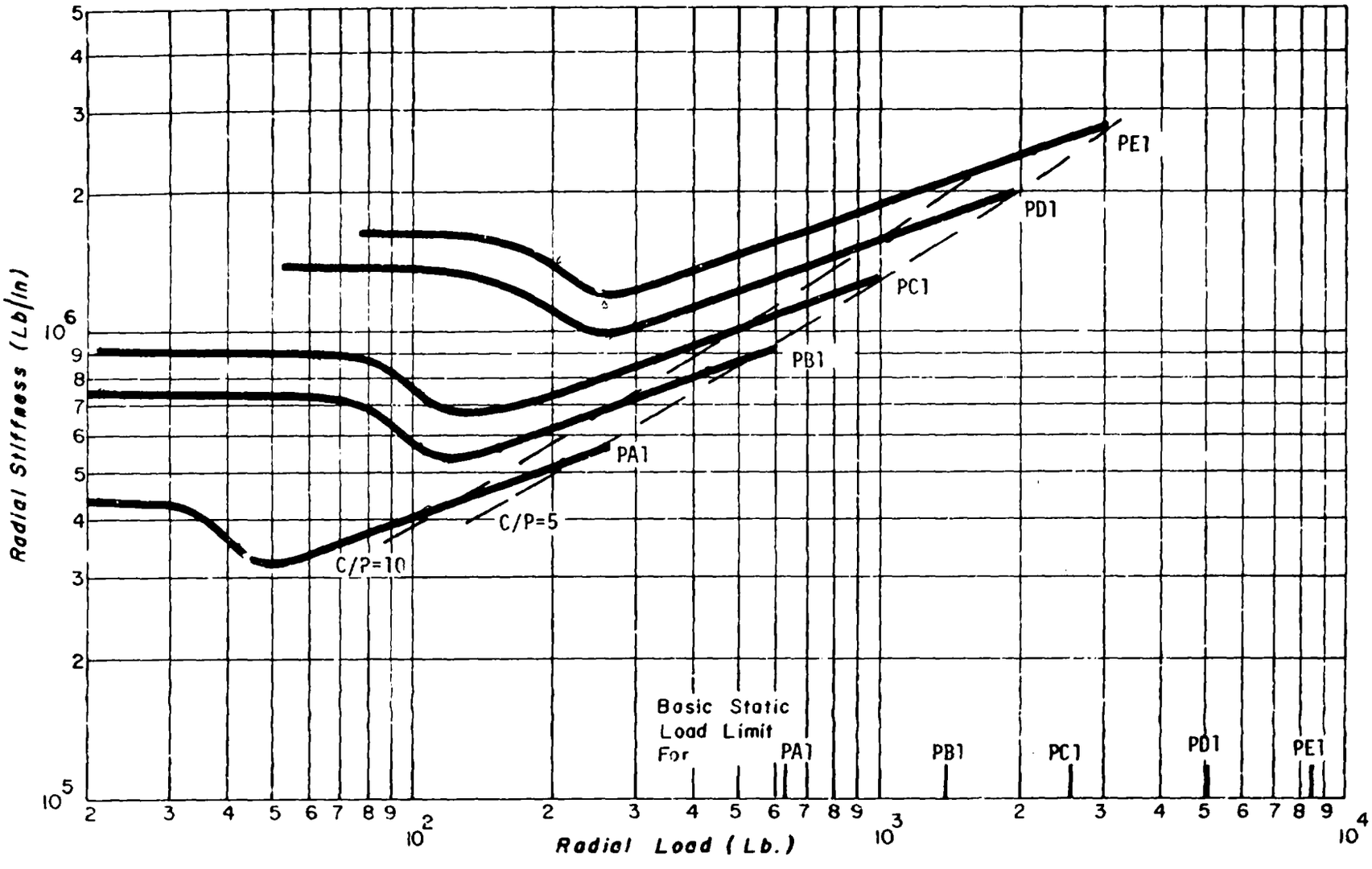

Fig. E-1 Radial Stiffness for Angular Contact Bearing, Preload -- Selected Light, $\beta = 15^{o}$

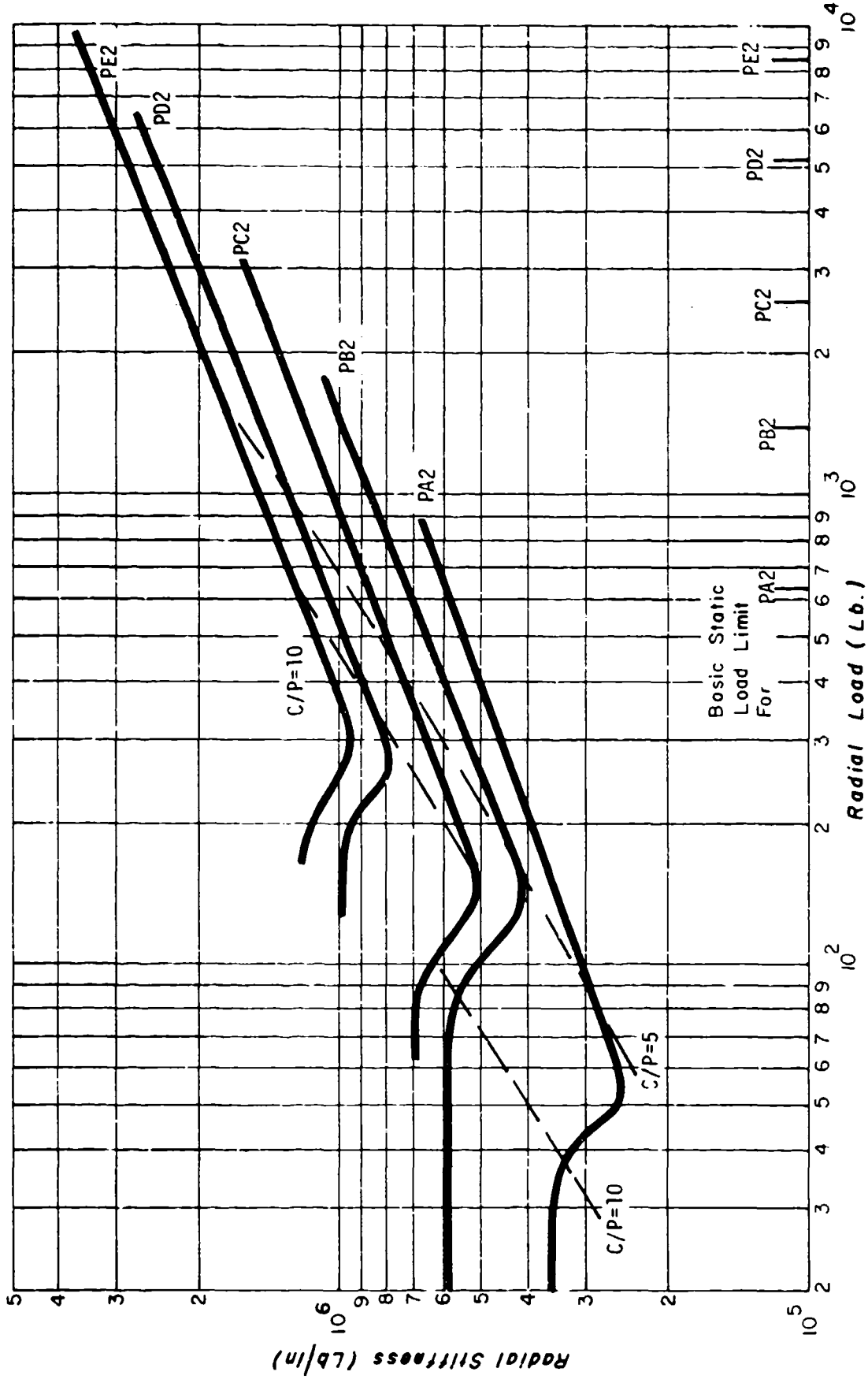

Fig. E-2 Radial Stiffness for Angular Contact Bearing, Preload -- Selected Light, $\beta = 15^{\circ}$

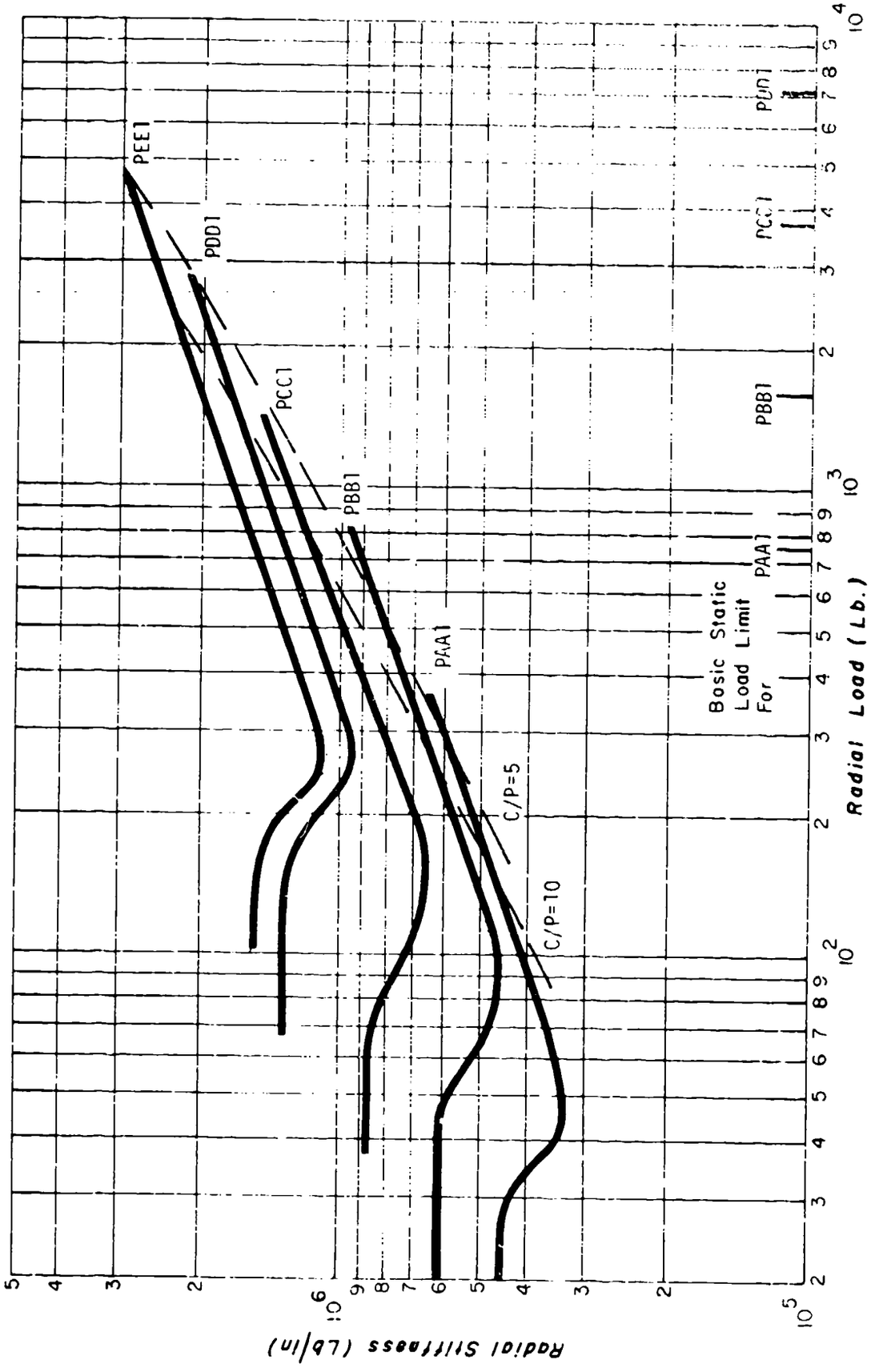

Fig. E-3 Radial Stiffness for Angular Contact Bearing, Preload -- Selected Light, $\beta = 15^{\circ}$

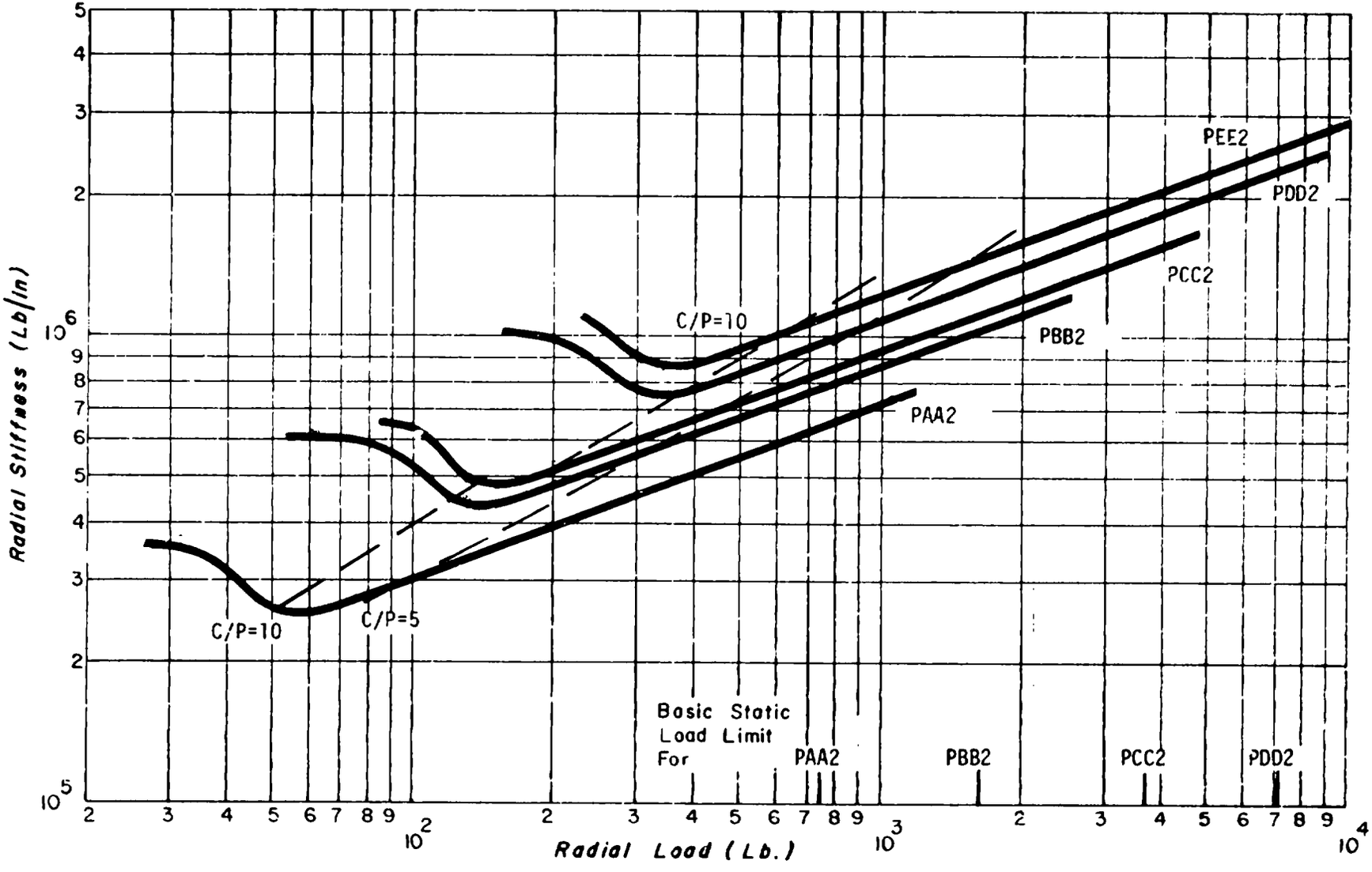

Fig. E-4 Radial Stiffness for Angular Contact Bearing, Preload -- Selected Light, $\beta = 15^{\circ}$

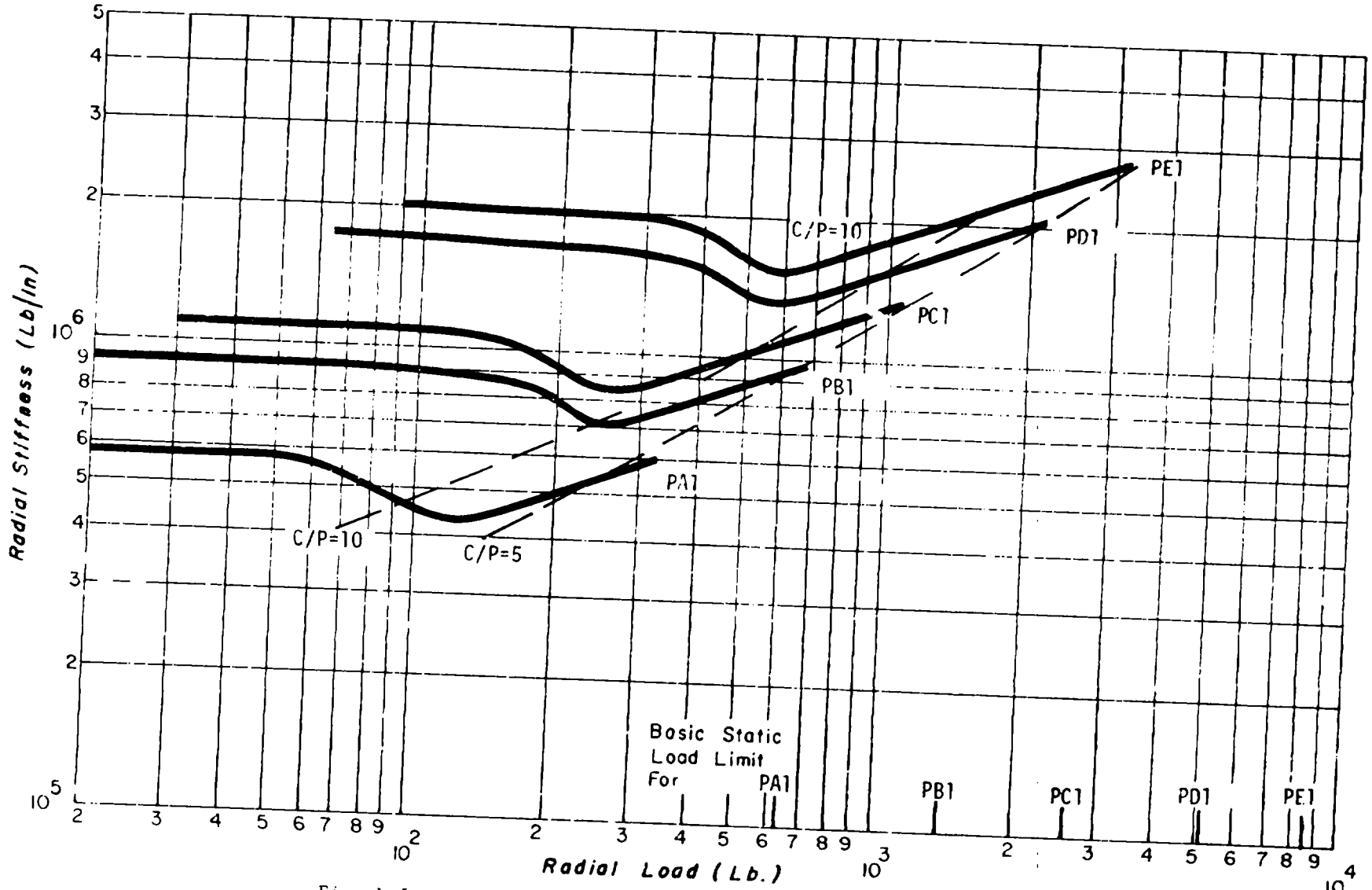

Fig. E-5 Radial Stiffness for Angular Contact Bearing, Preload -- Moderate, $\beta = 15^{\circ}$

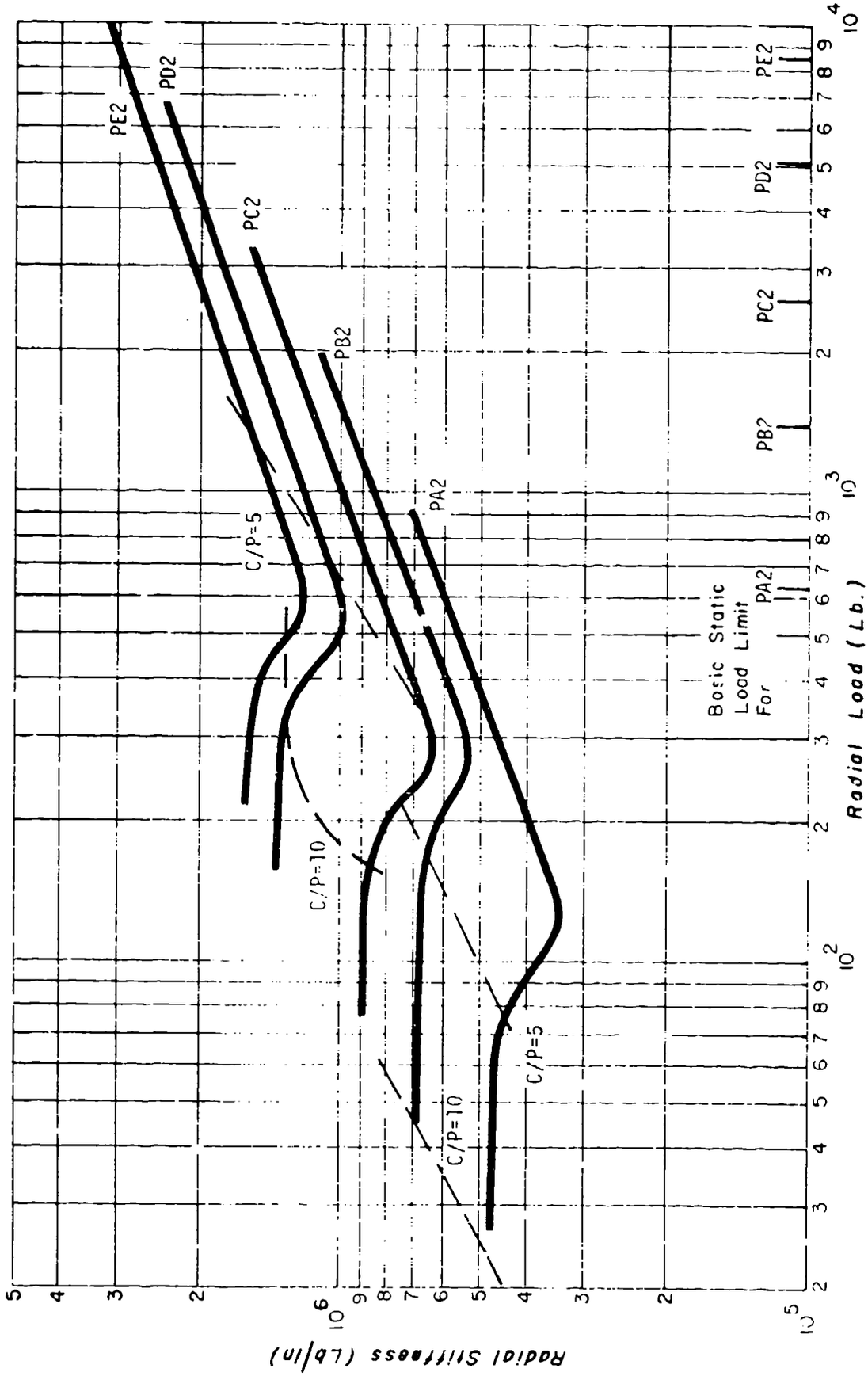

Fig. E-6 Radial Stiffness for Angular Contact Bearing, Preloaded -- Mounted

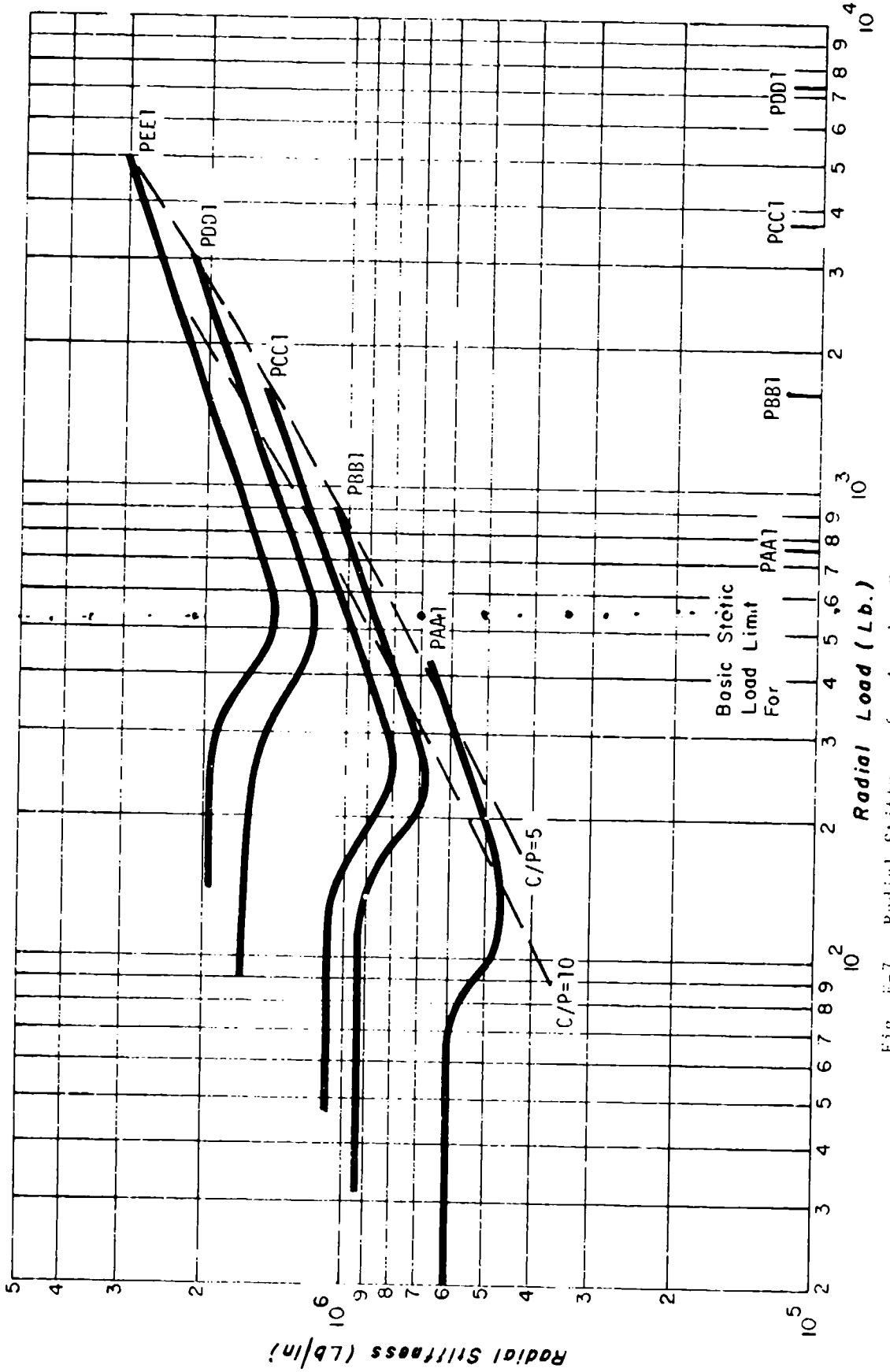

Fig. E-7 Radial Stiffness for Angular Contact Bearings, Preload -- Moderate, $\beta = 15^{\circ}$

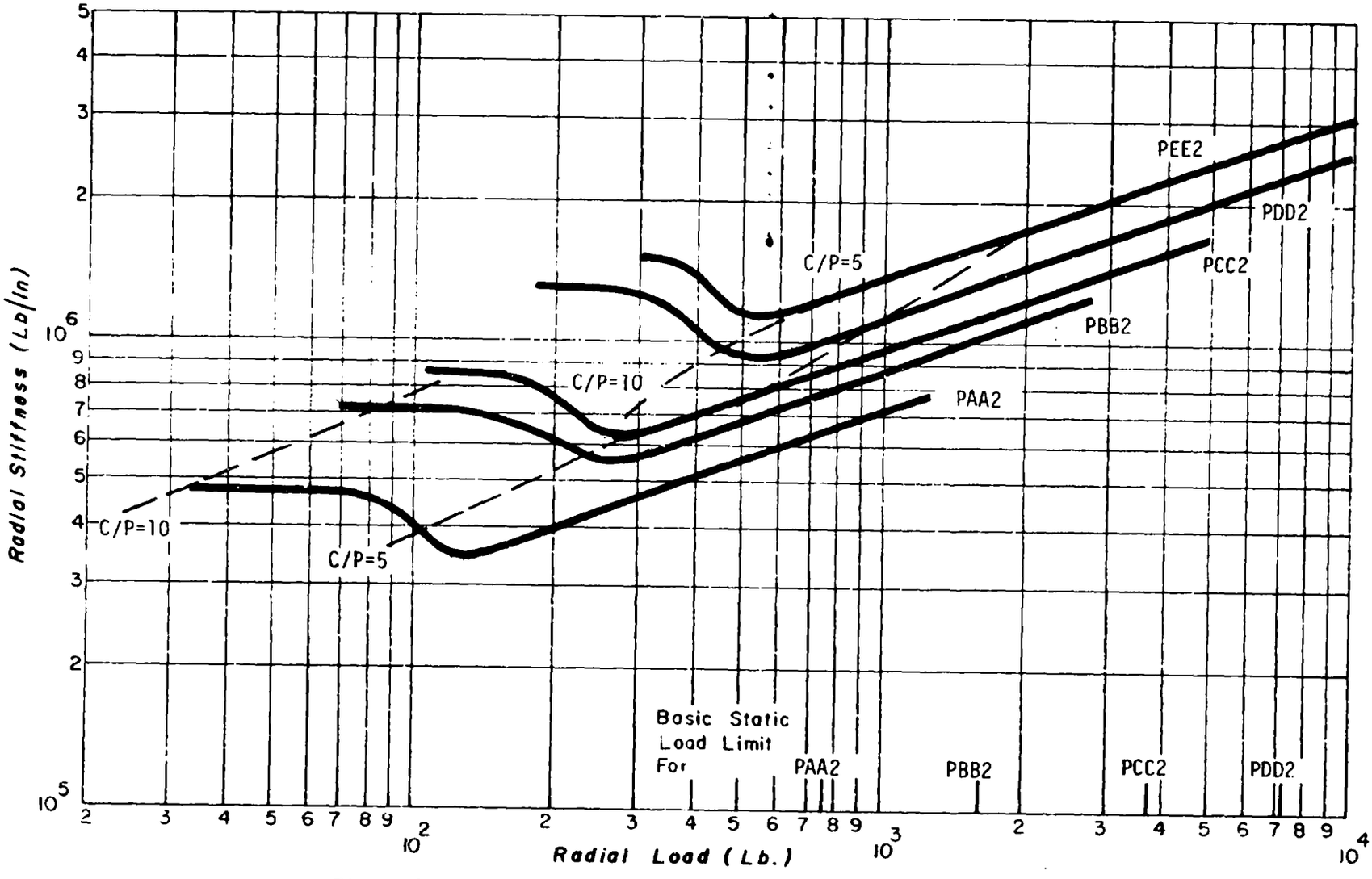

Fig. E-8 Radial Stiffness for Angular Contact Bearing,
Preload -- Moderate,
$\beta = 15^{\circ}$

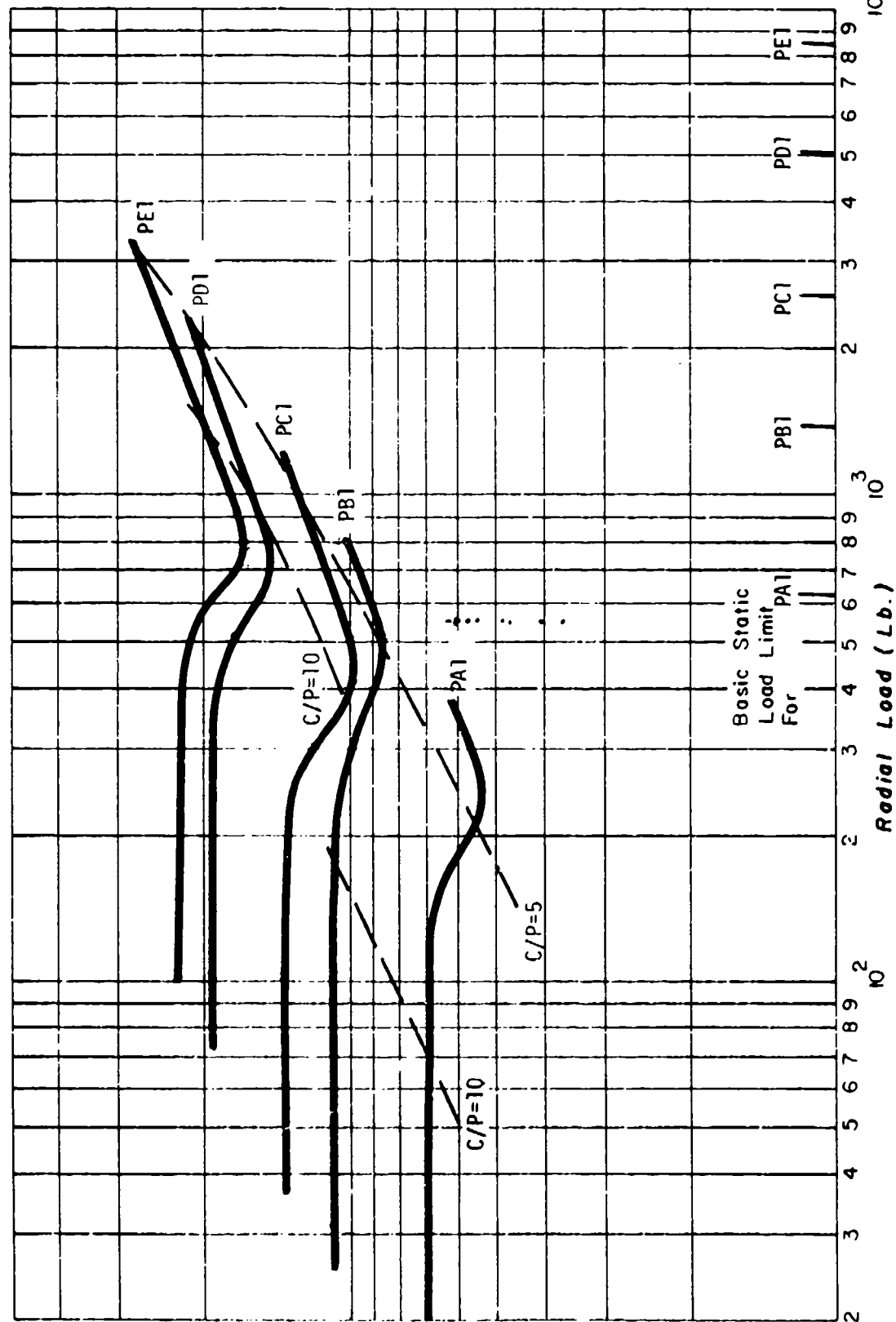

Fig. E-9 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, β - 15°

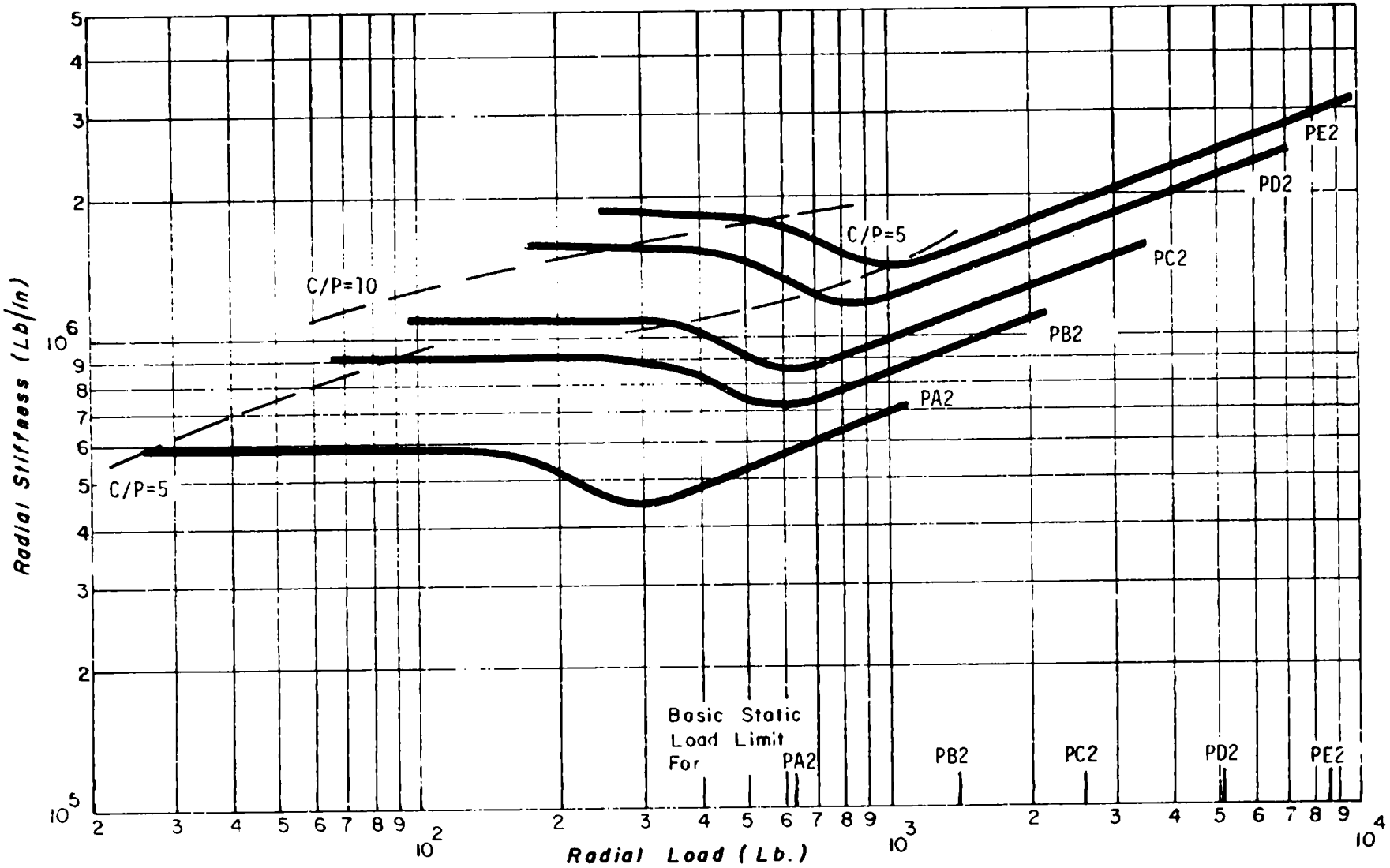

Fig. E-10 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\beta = 15°$

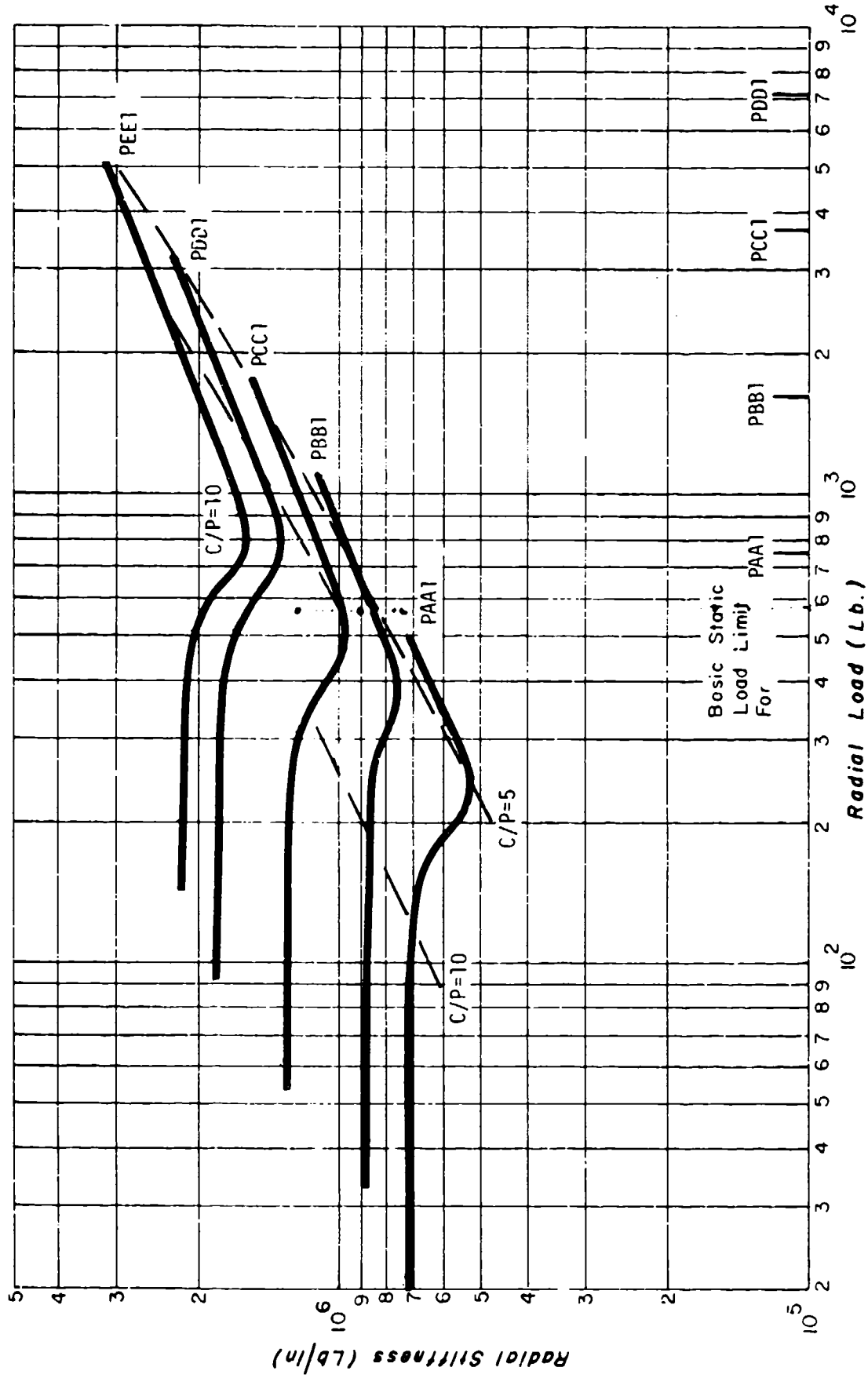

Fig. E-11 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\beta = 15^{\circ}$

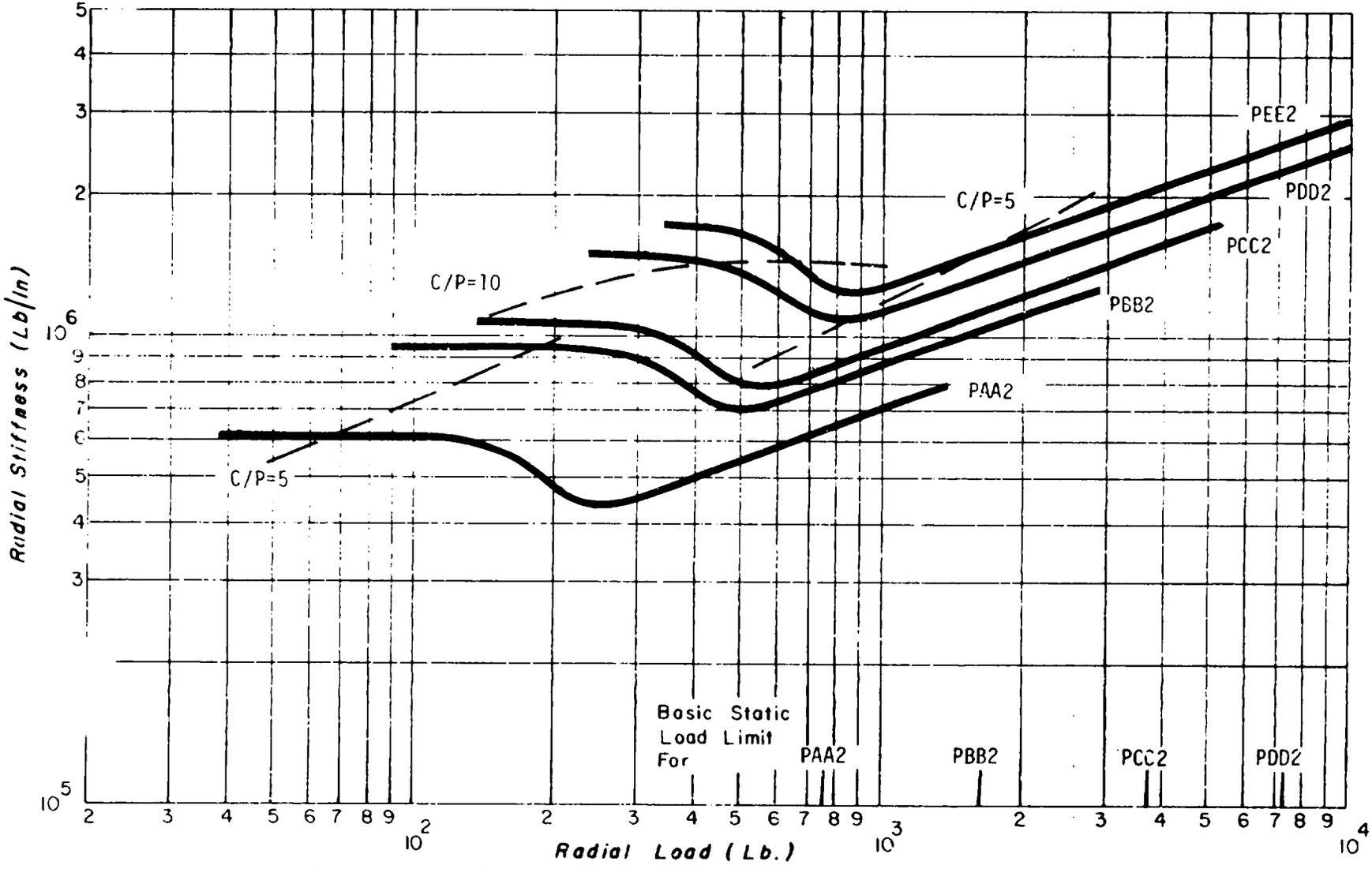

Fig. E-12 Radial Stiffness for Angular Contact Bearing, Preload -- Preferred Heavy, $\beta = 15^{\circ}$

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